Secondary Thermal Cracks in EGS: a Variational Approach

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We present a mechanistically faithful and mathematically sound approach to the numerical simulation of secondary thermal cracks propagation in EGS based on the *variational approach to fracture* [1-4]. While remaining compatible with classical theories, this approach provides a unified framework to crack nucleation, full path identification, including interaction between multiple cracks, branching or kinking in 2 and 3D. We present large scale numerical experiments, and compare our results with the literature.

1. Introduction:

The concept of Enhanced Geothermal System relies on the ability to artificially generate optimally connected fracture networks in hot dry rocks, maximizing heat transfer. A predictive understanding of the mechanisms used in creating a fracture network, of its evolution over time, and of its heat transfer properties is therefore essential to predicting and optimizing the performance of Enhanced Geothermal Systems.

As cold water is circulated through a reservoir, rock contract, creating tensile stresses, and potentially leading to the nucleation of new cracks, typically normal to existing fault lines. This effect is usually referred to as secondary thermal fracture. Because of the limited penetration of the heat front, secondary cracks usually do not penetrate very far into the reservoir, and the elastic energy release must be achieved through the nucleation of many short cracks. This mechanism can be taken advantage of in the setting of reservoir stimulation. In the longer term, it can also potentially lead to dramatic changes in the heat transfer characteristics of an existing reservoir.

2. The variational approach to fracture:

In classical approaches to quasi-static brittle fracture, the elastic energy restitution rate $G = -\partial E / \partial l$ induced by the infinitesimal growth of a single crack along an a priori know path (derived from the stress intensity factors) is compared to a critical energy rate G_c and propagation occurs when $G = G_c$, the celebrated Griffith criterion. The essence of the variational approach [4] is to recast the Griffith criterion in a variational setting, namely as the minimization over any crack set Γ (i.e. any curve in 2D or surface in 3D) and any kinematically admissible displacement field u of a total energy consisting of the sum of the stored potential elastic energy and a surface energy proportional to the length of the cracks in 2D or their area in 3D. This model can be formulated in a time continuous manner but for the sake of simplicity, we restrict our exposition to a discrete in time

formulfieldation: we consider a discretization of the time interval $(0,T)$ into N intervals $0 = t_0 < t_1 < \ldots < t_N = T$. We consider a body occupying a region Ω of space. On a part $\partial_d\Omega \subset \partial\Omega$, we prescribe a displacement $g(t)$ while the remaining part of $\partial\Omega$ is traction free. Then for any displacement and crack configuration (u,Γ) one defines the total energy

$$E(u,\Gamma) = \Pi(\mathbf{e}(u)) + G_c\,\mathrm{H}^{N-1}(\Gamma), \tag{1}$$

where $\Pi(\mathbf{e}(u))$ denotes the stored potential elastic energy of the considered system subject to a displacement u and cracked along Γ, $\mathrm{H}^{N-1}(\Gamma)$ denotes the N−1–dimensional Hausdorff measure of Γ, *i.e.* its length in 2D and its surface area in 3D, and $\mathbf{e}(u)$ the linearized strain field. We the depart from Griffith by postulating that at each time step t_i, the displacement and crack sets (u_i,Γ_i) are given by the global minimizers of E satisfying the boundary condition $u_i = g(t_i)$ on $\partial_d\Omega$, and under the irreversibility condition $\Gamma_i \supset \Gamma_{i-1}$. We insist that in this approach, the reliance on small increment of a single pre-existing crack growing along a pre-existing path of Griffith's classical theory has been fully replaced with global minimization. In particular, at each time step, nucleation of new cracks or changes in cracks topology are allowed, if they lead to a sufficient elastic energy release. We also note that the formulation applies identically to the two and three dimensional cases, and that evolutions computed through this approach satisfy Griffith's widely accepted criterion. Lastly, we refer to the wealth of literature dealing with such free discontinuity approaches from a mathematical, mechanical or numerical standpoint compiled in [4].

Numerical implementation

The numerical implementation of (1) is a challenging problem. The admissible displacement fields are discontinuous, but the location of their discontinuities is not known in advance, a requirement of many classical discretization methods. Also, the incremental fracture or surface energy term in (1) requires approximating the location of cracks, together with their length, a much more challenging issue (see the studies of anisotropy induced by the grid [5] and the mesh [6].)

The approach we present here is based on the variational approximation by elliptic functionals [2, 3]. A small regularization parameter ε is introduced and the location of the crack is represented by a smooth "crack regularization function" v taking values 0 close to the crack and 1 far from them. More precisely, one can prove (see [7] for instance) that as $\varepsilon \to 0$, the regularized energy

$$E_\varepsilon(u,v) = \int_\Omega v^2\Pi(\mathbf{e}(u))\,dx + G_c\int_\Omega \frac{(1-v)^2}{4\varepsilon} + \varepsilon|\nabla v|^2\,dx \tag{2}$$

approaches E in the sense of Γ–converges, and that the sequence of minimizers for E_ε is compact, which implies that the minimizers of E_ε converge as $\varepsilon \to 0$ to that of E.

The main feature of the regularized problem is that it does not require an explicit representation of the crack network. Instead all computations are carried out on a fixed mesh and the arguments of E_ε are smooth functions which can be approached using standard finite elements. Indeed, in the antiplane case, one can show that the restriction $E_{\varepsilon,h}$ of E_ε to a linear finite element space Γ–converges to E, provided that the mesh size h is such

that $h = o(\varepsilon)$. These two convergence results combined show that the solutions of the variational fracture model can be approximated by solving a linear finite elements discretization of (2) for "small" ε and h.

The actual minimization of (2) can be challenging, due to its stiffness and non-convexity. The key observation is that while it is not convex, the regularized energy is convex in each of the $(u,\bullet)$ and $(\bullet,v)$ directions. We make full use of this property by implementing an alternate minimizations algorithm in which the regularized energy is successively minimized with respect to u and v until convergence is reached. This approach is similar to a block Newton or a multi-physic preconditioner. It is globally convergent and each minimization step can be achieved using a fully implicit scheme, therefore avoiding the issue encountered while using explicit schemes on stiff problems.

The numerical implementation is based on a fully parallel object-oriented unstructured finite element library developed by the authors. By design, it separates unstructured mesh management, evaluation of the variational forms, discretization scheme (i.e. element types), and solvers. The mesh management and solver layers of the code are based on sieve [8]. The parallel linear algebra is implemented using PETSc [9, 10] while the scalable constrained and unconstrained optimization routines are provided by TAO [11].

3. A classical experiment

We illustrate the strength of our approach by revisiting a now classical experiment [12-14] dealing with convective cooling of a heated glass slab using a mechanistically faithful and mathematically variational formulation of brittle fracture which we extend to account for thermal cracks. We consider a rectangular glass slab of width W and height L with Hooke's law $\mathbf{A}$ associated with a Young's modulus E and a Poisson ratio ν. The thermal expansion tensor is α, the thermal diffusivity κ, and the temperature distribution T is such that $T(x,y,0) = T_0$. We assume that the glass is perfectly brittle, and behaves like linearly away from the cracks, i.e. that the elastic energy in (2) is given by

$$\Pi\big(\mathbf{e}(u),T\big) = \tfrac{1}{2}\int_\Omega \mathbf{A}\big(\mathbf{e}(u) - \alpha T\big) : \big(\mathbf{e}(u) - \alpha T\big)\, dx. \tag{3}$$

As the time scale for heat transfer is very large compared to the elastic wave speed in glass, we assume that at each time, the sample reaches an elastic equilibrium while the temperature field is given by transient heat transfer. We also neglect the effect of cracks on the heat transfer throughout the sample, and thermoelastic effects, *i.e.* assume that the deformation is slow enough that it induces no changes in the temperature fields. These assumptions, while technically necessary in our approach allow us to compare our numerical experiments with the experimental, numerical and analytical literature. In this setting, we simply compute the temperature field at each time step then minimize the total energy (2) modified to account for (3).

Heat transfer

At $t = 0$, the lower edge of the sample is brought in contact with dry ice with temperature T_s, and the temperature distribution for $t > 0$ is given by the transient heat equation:

$$\frac{\partial T}{\partial t} - \kappa \nabla^2 T = 0, \tag{4}$$

in the domain, while along the lateral and upper edges we assume null flux:

$$\frac{\partial T}{\partial n} = 0,$$

and Newtonian cooling along its lower edge:

$$\frac{\partial T}{\partial y} = \bar{h}\big((T(x,0,t) - T_s\big). \tag{5}$$

Here, $\bar{h} = hL/k$ denotes the Biot number, h being the heat transfer coefficient and k the thermal conductivity of the plate. In this setting, the solution of (4) depends only on the y coordinate and can be approximated (see for instance) by

$$T_h^{\infty}\big(y,t\big) = -(T_s - T_0)\mathrm{Erfc}(y/2\tau) - e^{(\bar{h}y+\bar{h}^2\tau^2)}\mathrm{Erfc}(y/2\tau + \bar{h}\tau) \tag{6}$$

where $\tau = \sqrt{t}$.

	p	a_0	τ_0	a_1	τ_1	a_2	τ_2	a_3	τ_3
Bahr &al	.13	.06	n/a	.14	.09	.35	.23	.45	.3
Numerical values	.15	.02	.02	.08	.05	.33	.16	.48	.25

Table 1: Comparison between numerical and analytical solutions.

Numerical experiments and comparison with the literature

Using classical dimension analysis, it is easy to see that up to a rescaling, our model can be reduced to a domain of unit height, unit Young's modulus, fracture toughness and thermal expansion coefficient, and normalized temperature $\beta\Delta T$, where β can be interpreted as the ratio between the elastic energy due to the thermal stress and the energy of a unit length fracture (see [12, 15] for instance). We focus on the case of a domain of width 5 under a normalized temperature contrast $\beta\Delta T = 54$ treated at length in [12] (case III). In this setting, a careful energy balance and stability analysis based on Griffith's criterion leads to the following: until a critical time τ_0, the sample remains intact, then a network of periodic cracks of length $a_0 \approx .06$ and spacing $p \approx .1285$ is nucleated. These cracks grow at the same speed until $\tau = \tau_1 \approx .09$ where the cracks reach a length $a_1 \approx .14$. At $\tau = \tau_1$, the fracture network has propagated deep enough that the thermal stresses are not sufficient to lead to propagate each crack tips, and half of the cracks stall. For $\tau_1 \le \tau \le \tau_2 \approx .225$, the same arrest mechanism activates once more at a crack length $a_2 \approx .335$. The remaining cracks continue growing until $\tau = \tau_3 \approx .3$ then stop upon reaching a final length $a_3 \approx .453$.

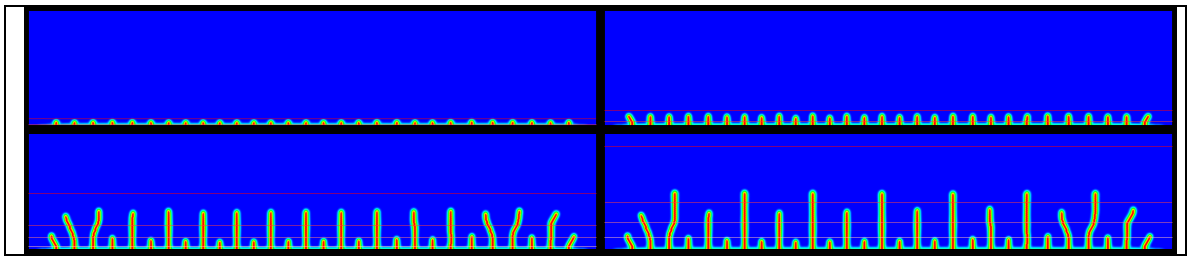

Figure 1: Convective heat transfer in a glass slab. Snapshots of the crack set and iso-temperatures at times *t*=5E-4, 2.25E-3, 2.5E-2, and 8.E-2.

We ran the same experiment using the method described above. Figure 1 represents the v–field representing the crack set and the level lines of the temperature field for τ =

τ0 (top left), $\tau = \tau_1$ (top right), $\tau = \tau_2$ (bottom left), and $\tau = \tau_3$ (bottom right). From a quantitative standpoint, the evolution is similar to that describe above: initiation of a network of cracks, propagation, arrest of half of the cracks and propagation of the remaining half, arrest of half of the remaining cracks, followed by propagation then arrest of the remaining ones. Again, we insist that no hypothesis was made on the crack network geometry. Instead, its periodic structure (up to boundary effects) comes up as the optimal crack geometry minimizing the total energy (2). Table 1 compares our numerical values with that of [12]. The quantitative agreement of the critical lengths a_2 and a_3 is excellent. The critical length a_0 and a1 are underestimated, leading to an underestimation of the critical times τ_0 and τ_1. This is due in part to the difficulty of accurately representing small cracks whose length is of the order of the mesh size in our approach.

Conclusions and extensions

The simple experiment presented above is a perfect illustration of our numerical approach and of its ability of accurately rendering crack nucleation, interaction between multiple cracks, and crack path. Figure 2 represents early numerical results for a three dimensional version of the same experiment in different domain geometry and temperature contrast. At this point, we have not yet been able to perform quantitative comparison between our numerical simulations and experiments. These numerical experiments serve as an illustration of the potential of our approach to deal with potentially complicated three-dimensional geometries.

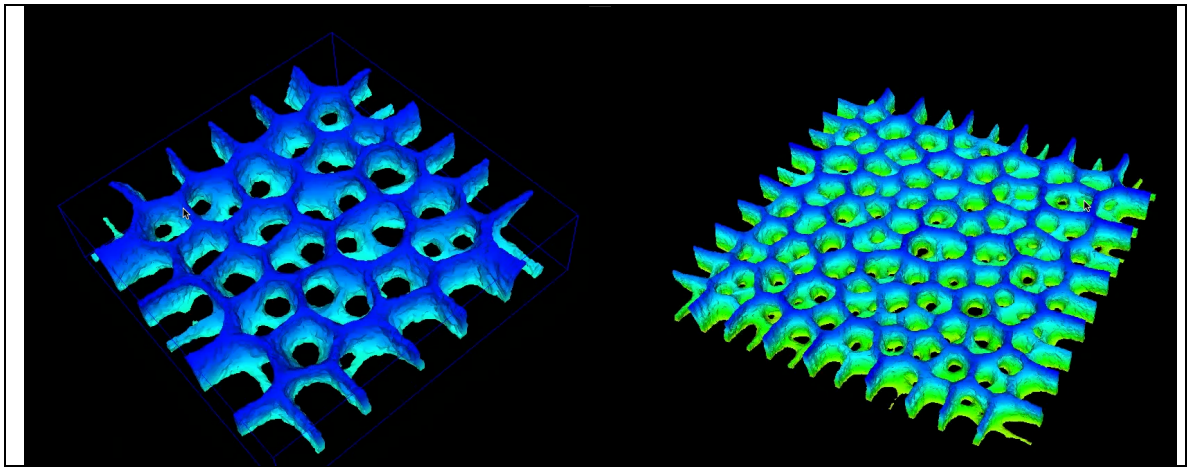

Figure 2: Convective heat transfer in a glass brick. Geometry of the final fracture system.

In all the computations presented above, we assume that the domain consist of a simple isotropic homogeneous material. Our approach can easily deal with material anisotropy or multiple materials. We have not yet investigated the role of heterogeneity on thermal cracks. We also have relied on a closed form solution of the heat equation in the domains. We are currently coupling transient heat transfer analysis and crack propagation. In doing so, we can harness the mathematical properties of the approximation of the fracture energy by a "phase–field"–like model and use the v–field to account for the effect of cracks on the heat transfer problem.

4. Acknowlegements

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