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NUMERICAL SIMULATION OF RESERVOIR STIMULATION - A VARIATIONAL APPROACH

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ABSTRACT

We present recent results on the modeling and numerical simulation of reservoir stimulation in Hot Dry Rocks. Our approach is based on a mechanistically faithful yet mathematically sound model, generalizing Griffith's idea of competition between bulk and surface energies. At each time increment, the fracture and displacement configuration of a reservoir is sought as the minimizer of a global energy. In doing so, the variational approach allows full crack path identification, interaction between multiple cracks, crack initiation and branching in two and three space dimensions. The numerical approach is based on building a regularized energy where cracks are represented by a smooth function, similar in spirit to a phase-field approach. In this paper, we focus on *thermal stimulation*.

INTRODUCTION

The concept of Enhanced Geothermal System relies on the ability to artificially generate optimally connected fracture networks in hot dry rocks, maximizing heat transfer. A predictive understanding of the mechanisms used in creating a fracture network, of its evolution over time, and of its heat transfer properties is therefore essential to predicting and optimizing the performance of Enhanced Geothermal Systems.

As cold water is circulated through a reservoir, rock contract, creating tensile stresses, and potentially leading to the nucleation of new cracks. Because of the difference of time scales required to achieve thermal and elastic equilibrium, it is reasonable to assume that at all time, the system achieves mechanical equilibrium, while thermal loads are modeled through transient heat transfer. In this framework, fracture nucleation may be initiated for small times (i.e. small penetration depth), and many short cracks may be necessary in order to achieve any elastic energy release. In short, thermal stimulation can possibly lead to the nucleation of many short cracks in relatively short times. In longer time scales, secondary thermal cracks may also appear during production and can potentially lead to dramatic changes in the heat transfer characteristics of a reservoir.

The mechanical modeling and numerical simulation of such a thermal stimulation requires a framework in which cracks nucleation, path identification –including complicated crack geometries–, interaction between existing and new cracks, must be properly accounted for in a three dimensional setting. In the following sections, we base our analysis on the mechanistically faithful yet mathematically rigorous variational formulation of brittle fracture (Francfort and Marigo, 1998, Bourdin et al. 2008). The numerical implementation is based on a concept of approximation by elliptic functional, similar to a phase-field model (Bourdin 2006, Bourdin 2007).

VARIATIONAL APPROACH TO FRACTURE

In classical approaches to quasi-static brittle fracture, the elastic energy restitution rate $G = -\partial E / \partial l$ induced by the infinitesimal growth of a single crack along an a priori know path (derived from the stress intensity factors) is compared to a critical energy rate G_c and propagation occurs when $G = G_c$, the celebrated Griffith criterion. The essence of the variational approach (Francfort and Marigo, 1998) is to recast the Griffith criterion in a variational setting, namely as the minimization over any crack set Γ (i.e.

any curve in 2D or surface in 3D) and any kinematically admissible displacement field u of a total energy consisting of the sum of the stored potential elastic energy and a surface energy proportional to the length of the cracks in 2D or their area in 3D. This model can be formulated in a time continuous manner but for the sake of simplicity, we restrict our exposition to a discrete in time formulation: we consider a discretization of the time interval (0,*T*) into *N* intervals $0 = t_0 < t_1 < \cdots < t_N = T$. We consider a body occupying a region Ω of space. On a part $\partial_d\Omega \subset \partial\Omega$, we prescribe a displacement $g(t)$ while the remaining part of $\partial\Omega$ is traction free. Then for any displacement and crack configuration (u, Γ) one defines the total energy

$$E(u,\Gamma) = \Pi(\mathbf{e}(u)) + G_c \mathrm{H}^{N-1}(\Gamma), \qquad (1)$$

where $\Pi(\mathbf{e}(u))$ denotes the stored potential elastic energy of the considered system subject to a displacement u and cracked along Γ, $\mathrm{H}^{N-1}(\Gamma)$ denotes the N–1–dimensional Hausdorff measure of Γ, *i.e.* its length in 2D and its surface area in 3D, and $\mathbf{e}(u)$ the linearized strain field. We then extend Griffith's ideas by postulating that at each time step t_i, the displacement and crack sets (u_i, Γ_i) are given by the global minimizers of E satisfying the boundary condition $u_i = g(t_i)$ on $\partial_d\Omega$, and the irreversibility condition $\Gamma_i \supset \Gamma_{i-1}$. We insist that in this approach, the reliance on small increment of a single pre-existing crack growing along a pre-existing path of Griffith's classical theory has been fully replaced with global minimization. In particular, at each time step, nucleation of new cracks or changes in cracks topology are allowed, if they lead to a sufficient elastic energy release. We also note that the formulation applies identically to the two and three-dimensional cases, and that evolutions computed through this approach satisfy Griffith's widely accepted criterion. For a more in-depth presentation of the variational approach to brittle fracture, we refer to the references previously cited.

Numerical implementation

The numerical implementation of (1) is a challenging problem. The admissible displacement fields are discontinuous, but the location of their discontinuities is not known in advance, a requirement of many classical discretization methods. Also, the incremental fracture or surface energy term in (1) requires approximating the location of cracks, together with their length, a much more challenging issue. In particular, special consideration needs to be given to the possible anisotropy induced by the grid (Chambolle 1999) or the mesh (Negri, 1999).

The numerical approach we present here is based on the variational approximation by elliptic functionals originally proposed by Ambrosio and Tortorelli, (1990) in the context of image segmentation and extended to brittle fracture (Bourdin et al 2000, Bourdin 2007). A small regularization parameter ε is introduced and the location of the crack is represented by a smooth "crack regularization function" v taking values 0 close to the crack and 1 far from them. More precisely, one can prove (see the references cited above) that as $\varepsilon \to 0$, the regularized energy

$$E_\varepsilon(u,v) = \int_\Omega v^2 \Pi(\mathbf{e}(u))\, dx + G_c \int_\Omega \frac{(1-v)^2}{4\varepsilon} + \varepsilon |\nabla v|^2 \, dx \qquad (2)$$

approaches E in the sense of Γ–converges, and that the sequence of minimizers for E_ε is compact, which implies that the minimizers of E_ε converge as $\varepsilon \to 0$ to that of *E*.

The main feature of the regularized problem is that it does not require an explicit representation of the crack network. Instead all computations are carried out on a fixed mesh and the arguments of E_ε are smooth functions, which can be approached using standard finite elements. Indeed, in the anti-plane case, one can show that the restriction $E_{\varepsilon,h}$ of E_ε to a linear finite element space Γ–converges to E, provided that the mesh size h is such that $h = o(\varepsilon)$. These two convergence results combined show that the solutions of the variational fracture model can be approximated by solving a linear finite elements discretization of (2) for "small" ε and h.

The actual minimization of (2) can be challenging, due to its stiffness and non-convexity. The key observation is that while it is not convex, the regularized energy is convex in each of the $(u, \bullet)$ and $(\bullet, v)$ directions. We make full use of this property by implementing an alternate minimizations algorithm in which the regularized energy is successively minimized with respect to *u* and *v* until convergence is reached. This approach is similar to a block Newton or a multi-physic preconditioner. It is globally convergent and each minimization step can be achieved using a fully implicit scheme, therefore avoiding the issue encountered while using explicit schemes on stiff problems.

The numerical implementation is based on a fully parallel object-oriented unstructured finite element library developed by the authors. By design, it separates unstructured mesh management, evaluation of the variational forms, discretization scheme (i.e. element types), and solvers. The mesh management and solver layers of the code are based on the Sieve framework (Knepley and Karpeev 2009). The parallel linear algebra is implemented using PETSc (Balay et al. 1997, 2009, 2010) while the scalable constrained and unconstrained optimization routines are provided by TAO (Benson et al. 2007).

A 2D EXAMPLE

We illustrate the strength of our approach by revisiting a now classical convective cooling of a heated glass slab experiment described in depth in Geyer and Nemat-Nasser (1982), Bahr et al. (1988), or Tester (1989) for instance.

We consider a rectangular glass slab of width W and height L with Hooke's law $\mathbf{A}$ associated with a Young's modulus E and Poisson ratio ν, and thermal expansion coefficient α. The sample's initial temperature is $T(x,y,0)=T_0$ and at $t=0$, its lower edge is brought in contact with dry ice held at temperature T_s. In this setting, the temperature profile can be computed explicitly, assuming null flux through the lateral and superior edges of the domain, and when the heat penetration depth is small compared to L.

As the time scale for heat transfer is large compared to the elastic wave speed in glass, we make the usual assumption that at each time the sample reaches an elastic equilibrium, while the temperature field is given by transient heat transfer. We also neglect the effect of cracks on the heat transfer throughout the sample, and thermoelastic effects, *i.e.* assume that the deformation is slow enough that it induces no changes in the temperature fields. These assumptions, while not technically necessary in our approach allow us to compare our numerical experiments with the experimental, numerical and analytical literature. In this setting, we simply compute the temperature field at each time step, then minimize the total energy in which thermal expansion is accounted for by replacing the elastic potential $\Pi(\mathbf{e}(u))$ in (1) or (2) with

$$\Pi(\mathbf{e}(u),T)=\tfrac{1}{2}\int_\Omega \mathbf{A}(\mathbf{e}(u)-\alpha T):(\mathbf{e}(u)-\alpha T)\,dx. \quad (3)$$

Numerical experiments, and comparison with analytical solutions

Using classical dimension analysis, it is easy to see that up to a rescaling, our model can be reduced to a domain of unit height, unit Young's modulus, fracture toughness and thermal expansion coefficient, and normalized temperature $\beta\Delta T$, where β can be interpreted as the ratio between the elastic energy due to the thermal stress and the energy of a unit length fracture (see Bahr et al. 1988 or Jenkins 2005, for instance). As often in heat transfer problem, we consider a characteristic time scale $\tau=\sqrt{t}$. We first focus on the case of a domain of width 5 under a normalized temperature contrast $\beta\Delta T=54$ treated at length in Bahr et al. (1988), case III.

In this setting, a careful energy balance and stability analysis based on Griffith's criterion leads to the following: until a critical time τ_0, the sample remains intact, then a network of periodic cracks of length $a_0\approx.06$ and spacing $p\approx.1285$ is nucleated. These cracks grow at the same speed until $\tau=\tau_1\approx.09$ where the cracks reach a length $a_1\approx.14$. At $\tau=\tau_1$, the fracture network has propagated deep enough that the thermal stresses are not sufficient to lead to propagate each crack tips, and half of the cracks stall. For $\tau_1\le\tau\le\tau_2\approx.225$, the same arrest mechanism activates once more at a crack length $a_2\approx.335$. The remaining cracks continue growing until $\tau=\tau_3\approx.3$ then stop upon reaching a final length $a_3\approx.453$. In a first set of experiment, we tried to prescribe the fracture network geometry by using a Cartesian mesh and a regularization parameter ε of the order of a fraction of the mesh size, a choice known to lead to mesh dependency.

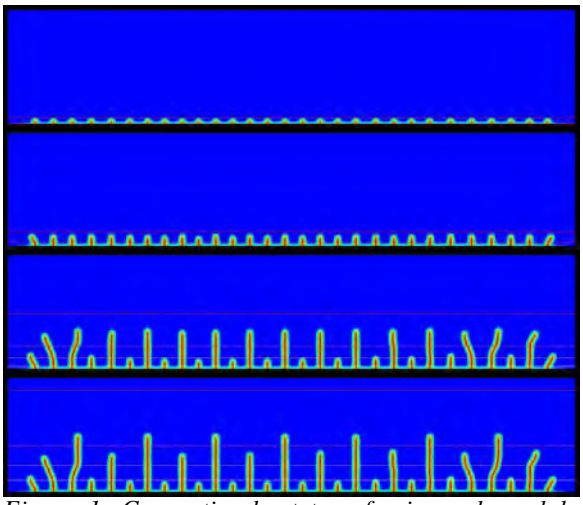


Figure 1: Convective heat transfer in a glass slab. Snapshots of the crack set and isotemperatures for $\tau=\tau_0,\tau_1,\tau_2,\tau_3$.

Figure 1 represents the v–field representing the crack set and the level lines of the temperature field for $\tau=\tau_0,\tau_1,\tau_2,\tau_3$ (top to bottom). From a quantitative standpoint, the evolution is similar to that describe above: initiation of a network of cracks, propagation, arrest of half of the cracks and propagation of the remaining half, arrest of half of the remaining cracks, followed by propagation then arrest of the remaining ones. Again, we insist that while we tuned our approximation to favor straight cracks, no other hypothesis was made on the crack network geometry. Its periodic structure (up to boundary effects) and the successive crack parking come up as the optimal crack geometry minimizing the total energy (2).

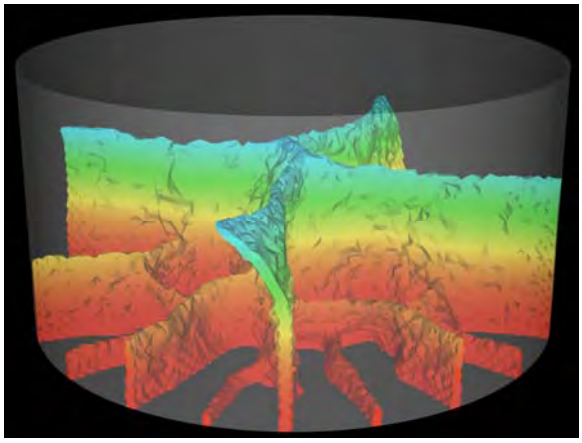
Table 1: Comparison between numerical and analytical solutions.

	p	a_0	τ_0	a_1	τ_1	a_2	τ_2	a_3	τ_3
Bahr	.13	.06	n/a	.14	.09	.35	.23	.45	.3
BMK	.15	.02	.02	.08	.05	.33	.16	.48	.25

Table 1 compares our numerical values with that of Bahr et al. (1988). The quantitative agreement of the critical lengths a_2 and a_3 is excellent. The critical length a_0 and a1 are underestimated, leading to an underestimation of the critical times τ_0 and τ_1. This is due in part to the difficulty of accurately representing small cracks whose length is of the order of the mesh size in our approach. Another factor is the difficulty to properly capture the critical bifurcation between stable branches of the total energy (2) in the case of nucleation of cracks with non-zero length. This issue can be in parts alleviated through the use of a backtracking algorithm (Bourdin 2007a), but this was not done here.

CONCLUSIONS AND EXTENSIONS

The simple experiment presented above is a perfect illustration of our numerical approach and of its ability of accurately rendering crack nucleation, interaction between multiple cracks, and crack path. Figures 2 and 3 represent early numerical results for a three dimensional versions of the same experiment for two domain geometries. These numerical experiments serve as an illustration of the potential of our approach to deal with potentially complicated three-dimensional geometries.



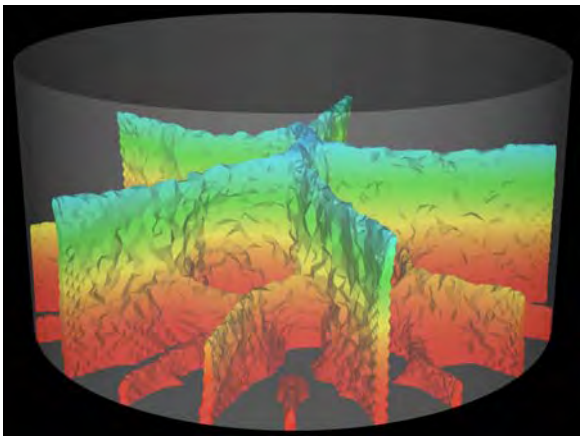
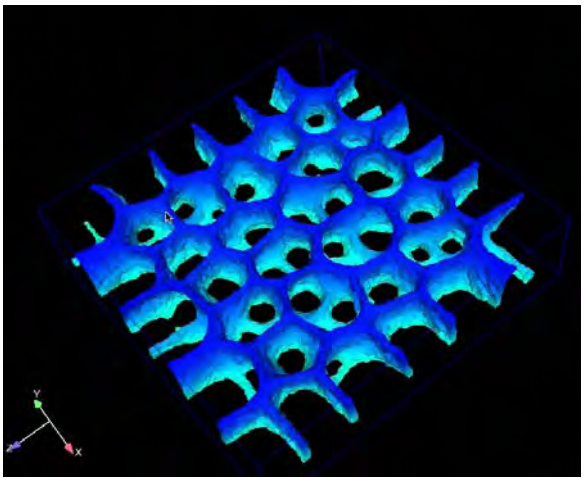


Figure 2: A three dimensional version of the experiment above highlights the potential complexity of crack patterns 3D, and the challenge in devising a suitable fracture model and numerical implementation.

At this point, we have not yet been able to perform quantitative comparison between our numerical simulations and experiments. From a qualitative standpoint, we notice that as the computational domain becomes large compared to the fracture propagation depth and spacing, network of polygonal cells are created, not unlike that of drying basaltic columns. In Figure 3, we performed computations on brick shaped domain. The material properties and experimental conditions are similar, but the domain size along each coordinate axis were doubled. Global properties of the fracture network, such as the average cell size and penetration depth remained unchanged. We are now investigating the quantitative comparison of these fracture networks with experiments, in particular with the scaling properties given in Bahr et al. (2010).



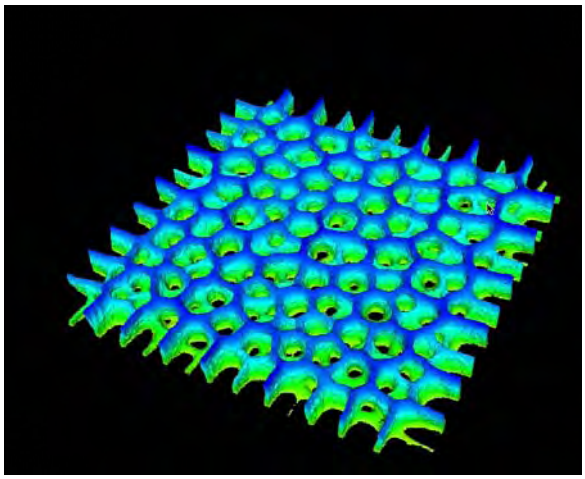


Figure 3: Numerical experiment on parallelepiped. As the domain size is changed, the critical fracture cell size and penetration depth appears to remain unchanged.

In all the computations presented above, we assume that the domain consist of a simple isotropic homogeneous material. Our approach can easily deal with material anisotropy or multiple materials. We have not yet investigated the role of heterogeneity on thermal cracks. We also have relied on a closed form solution of the heat equation in the domains. We are also currently coupling our fracture model with a reservoir simulator.

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