# HOMEWORK #1

Due: November 29 (Thursday) by midnight

**Instructions:**

- The assignment consists of *three* questions, worth respectively 4, 2 and 4 points.
- Submit your assignment *electronically* (via Email) to the instructor; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/CES717/template.m` (see also the link in the "Computer Programs" section on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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Using the vortex codes discussed during the lectures as a starting point, develop your own code that will accomplish the following tasks:

- compute the Hamiltonian $\mathcal{H}$ of the vortex system given by

$$\mathcal{H}(\mathbf{x}_1,\dots,\mathbf{x}_N) = -\frac{1}{4\pi}\sum_{\substack{i,j\\ i\neq j}}^{N}\Gamma_i\Gamma_j\ln|\mathbf{x}_i-\mathbf{x}_j|,$$

  where $\mathbf{x}_k = [x_k, y_k]$ is the position of the $k$–th vortex,
- implement the Adams–Bashforth scheme for integration of the particle trajectories; the Adams–Bashforth scheme is given by the formula $y^{n+1} = y^n + \frac{\Delta t}{2}[3f(y^n,t^n) - f(y^{n-1},t^{n-1})]$,
- implement regularization of the Biot–Savart kernel using the mollifier

$$\omega_\delta(r) = \frac{1}{2\pi\delta^2}\exp\left(-\frac{r^2}{2\delta^2}\right). \qquad (1)$$

Using the initial vortex distribution corresponding to the leapfrogging vortices (see the code `vortex_02.m`), perform computations in the following configurations:

1. no regularization of the Biot–Savart kernel and
   (a) Euler explicit time stepping,
   (b) Adams–Bashforth time stepping,
   (c) leapfrog time stepping;
2. leapfrog time stepping and kernel regularization using mollifier (1) with the following values of the cut–off $\delta = 1.0, 0.25$.

For both sets of computations plot

- the particle trajectories,
- the time–histories of the Hamiltonian $\mathcal{H}$,

obtained in the different cases. Use the time step $\Delta t = 5\cdot 10^{-2}$ and solve the equations over $N_t = 5\cdot 10^2$ time steps.