TEST #1

13:30–14:20, November 15 (Thursday) in HH/207

Make sure to put your name and ID number in the top–left corner of the answer sheet

No textbooks or notes allowed!

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1. Consider a problem in celestial dynamics in which you are given N point masses $\{m_1, \ldots, m_N\}$ in an unbounded three–dimensional domain.

   (a) Using the "particle–particle" (PP) formalism, state the complete system of equations describing the evolution of this system,

   (b) how will this description be modified in the "particle–mesh" (PM) formalism?

   (c) what is the total number of independent degrees of freedom in this problem?

   [2 points]

2. Consider a problem in hydrodynamics in which you are given N point vortices with circulations $\{\Gamma_1, \ldots, \Gamma_N\}$ in an unbounded two–dimensional domain.

   (a) Using the "particle–particle" (PP) formalism, state the complete system of equations describing the evolution of this system,

   (b) how will this description be modified in the "particle–mesh"(PM) formalism?

   (c) what is the total number of independent degrees of freedom in this problem?

   [2 points]

3. There is a vortex with positive (i.e., counter–clockwise) circulation Γ above a horizontal plate of infinite extent. Assuming that the vortex is a distance h away from the boundary, determine the velocity V of the vortex. Will the vortex be moving to the left, or to the right?
   HINT — Use the method of images to ensure that the "no–flow" velocity boundary conditions are satisfied at the plate; the complex velocity induced at $z = x + iy$ by a point vortex located at $z_0 = x_0 + iy_0$ is $V(z) = (u - iv)(z) = \frac{\Gamma}{2\pi 1} \frac{1}{z - z_0}$.
   [2 points]

4. Characterize the *global* order of accuracy and the stability properties of the following schemes for integration of ODEs:

   (a) Euler explicit,

   (b) Euler implicit,

   (c) Crank–Nicolson,

   (d) leapfrog.

   Which of them has the best conservation properties?
   [2 points]