TEST #1

13:30–14:20, September 27 (Thursday) in HH/207

Make sure to put your name and ID number in the top–left corner of the answer sheet

No textbooks or notes allowed!

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1. Given the following two weak formulations of Boundary Value Problems

   (a)
   $$u \in H^1(\Omega) \qquad \int_\Omega \nabla u \cdot \nabla v \, d\Omega = \int_\Omega f v \, d\Omega \qquad \forall_{v \in H^1(\Omega)},$$

   (b)
   $$u \in H_0^1(\Omega) \qquad \int_\Omega \nabla u \cdot \nabla v \, d\Omega = \int_\Omega f v \, d\Omega \qquad \forall_{v \in H_0^1(\Omega)},$$

   what are the corresponding strong formulations?
   [2 points]

2. Propose a weak formulation $a(u,v) = l(v),\ \forall_{v \in V}$ for the Boundary Value Problem

   $$\begin{cases} \dfrac{d^4u}{dx^4} = f \quad \text{in } [a,b], \\ u\big|_a = u\big|_b = 0, \\ \left.\dfrac{d^2u}{dx^2}\right|_a = \left.\dfrac{d^2u}{dx^2}\right|_b = 0. \end{cases}$$

   HINT: note that the form $a(u,v)$ must be *symmetric*.
   [2 points]

3. Assume V is an infinite–dimensional space and W is a finite–dimensional approximating space such that $W \subset V$ and $u \in V$. Let $\{w_1, \ldots, w_n\}$ be an *orthonormal* basis of W. Derive an expression for the coefficients $\{\alpha_i\}_{i=1}^n$ of the best approximation $w = \sum_{i=1}^n \alpha_i w_i$ of u. What can we say about the approximation error $u - w$ if $u \in W$?
   [2 points]

4. Your are given a grid shown in Fig. 1 which is to be used to solve a second–order Boundary Value Problem with boundary conditions as indicated in the Figure. Assuming piecewise linear interpolating functions and indexing of triangles and vertices as discussed at the lecture (i.e., going from the bottom up and from the left to the right in a given row)

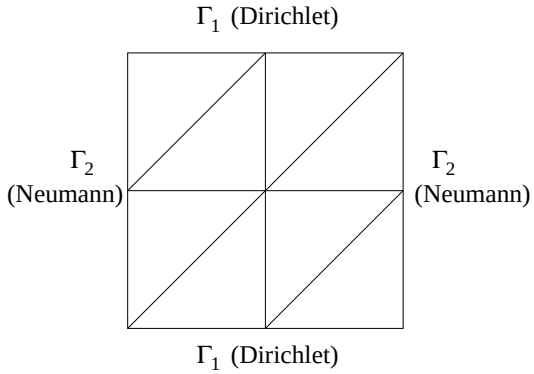

Figure 1: Grid and boundary conditions.

(a) determine the number N_f and the indices $f_1, \ldots, f_{N_f}$ of the *free nodes*,

(b) determine the number N_c and the indices $c_1, \ldots, c_{N_c}$ of the *constrained nodes*,

(c) using the table below determine which entries of the stiffness matrix $\mathbb{K}$ will vanish ($\bigcirc$) and which will be nonzero ($\surd$); leave the rows and columns corresponding to the constrained nodes blank.

	1	2	3	4	5	6	7	8	9
1									
2									
3									
4									
5									
6									
7									
8									
9									

[2 points]