HOMEWORK #3

Due: November 15 (Wednesday) by 5pm

Instructions:

- The assignment consists of *two* questions, each worth 5 points.
- Submit your assignment *electronically* (via Email) to the address `math2e03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH2E03/template.m` (see also the link in the "Computer Programs" section on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

---

1. Consider the Lotka–Volterra model *with fishing* given by the system of equations

$$\begin{cases} \dfrac{du}{dt} = u\left(1 - v - \delta'\right), \\ \dfrac{dv}{dt} = \alpha v\left(u - 1 - \dfrac{\delta'}{\alpha}\right), \end{cases} \tag{1}$$

where $\alpha = \frac{1}{2}$ and $\delta' = \frac{1}{2}$ and the initial conditions are chosen to correspond to the equilibrium solution (with fishing). Write a MATLAB code that will simulate a solution of this system in the time interval $[0,50]$ assuming that fishing takes place *only* when $20 \le t \le 30$. Ensure that system (1) is solved numerically with an accuracy of 10^{-6} (both absolute and relative) and plot

(a) solutions $u(t)$ and $v(t)$ as functions of time t,

(b) system trajectory in the (u,v) plane (the "phase portrait").

Hint — you should use the MATLAB instruction `if (condition) ... else ... end;` in the definition of a new modified right–hand side function
(5 points)

2. Consider the equation

$$u'' - \mu(1-u^2)u' + u = 0 \tag{2}$$

known as the van der Pol oscillator. Assuming that $\mu = 1.0$ and the initial conditions are $u(0) = 0$ and $u'(0) = 0.1$, write a MATLAB code that will solve equation (2) in the time interval $[0,40]$ and

(a) plot the phase portrait and vector field in the (u,u') coordinates (assume that $-3.0 \le u,u' \le 3.0$ when plotting the vector field),

(b) using the symbolic toolbox calculate the linearization of equation (2) around the equilibrium at the origin (0.0) and determine numerically its eigenvalues; what is the type of this equilibrium and is it structurally stable?

(c) solve numerically the linearized system in the time interval $[0,7]$ and using the same initial conditions as in point (a); plot the trajectory in the same figure as in point (a);

Hint — at the beginning convert the second–order equation (2) to a system of two first–order differential equations.
(5 points)