

**TEST #1**

9:30 — 10:20am, October 13, 2006

DR. B. PROTAS

LAST (FAMILY) NAME: \_\_\_\_\_

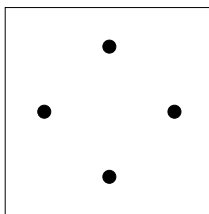
FIRST (GIVEN) NAME: \_\_\_\_\_

STUDENT NUMBER: \_\_\_\_\_

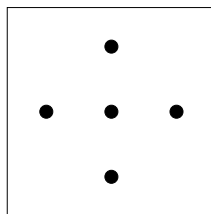
STUDENT MARK: \_\_\_\_\_ (Max 10 points)

- The test has 5 questions, each worth 2 points; please provide your answers on the reverse side (in question 2 you may use the diagrams provided on this side); you may also attach as many additional sheets as need, but make sure to label them clearly;
- Time allowed: **50 minutes**
- Only the McMaster Standard Calculator Casio FX991MS is allowed

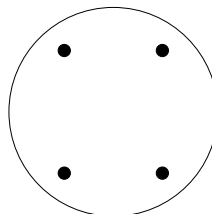
1. If the probability density function of a random variable  $x$  on the interval  $[1, e]$  is  $f(x) = \frac{k}{x}$ , where  $k$  is a constant, then what is the probability that  $1 \leq x \leq 2$ ? What is the expected value?
2. Sketch the Voronoi diagrams for the following sets of points briefly indicating how you construct them



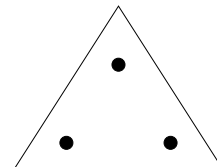
(a)



(b)

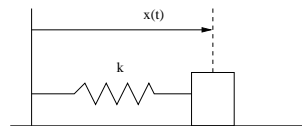


(c)



(d)

3. You are given an object with mass  $m$  sliding without friction on a flat surface. There is a spring attached to the object which exerts a force  $F = -kx(t)$ , where  $k > 0$  is constant and  $x(t)$  is the horizontal displacement of the object (see Figure). Derive the differential equation describing the displacement  $x(t)$  of the body. What would be appropriate initial conditions?



4. A principal is compounded quarterly with a nominal interest rate  $r$ . Find an expression for the difference  $r_{eff} - r$  between the yearly *effective* interest rate  $r_{eff}$  and  $r$  in terms of  $r$  itself and the frequency of compounding.
5. Mortgage compounded continuously is characterized by the differential equation  $P'(t) - \rho P(t) = -x$ , where  $\rho$  and  $x$  are a given interest rate and rate of payment (both assumed constant in time). Knowing that the initial value of the mortgage is  $P_0$ , find how the mortgage value  $P(t)$  varies with time  $t$ .