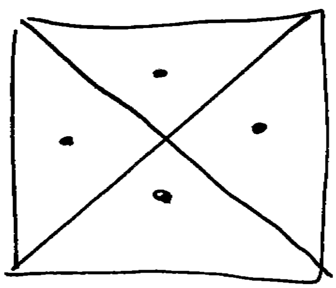
1) $\int_1^e \frac{k}{x}dx = k \ln x \Big|_1^e = k(\ln e - \ln 1) = k(1-0) = 1 \Rightarrow \underline{k=1}$

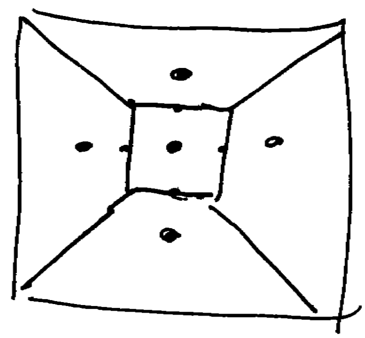
$\int_1^2 \frac{dx}{x} = \ln x \Big|_1^2 = \underline{\underline{\ln 2}} = P(1 \leq x \leq 2)$

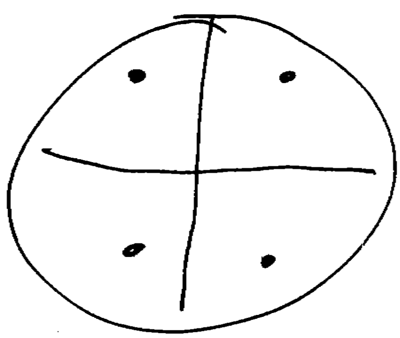
The expected value:

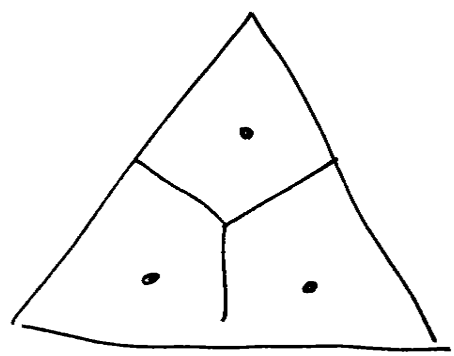
$E(x) = \int_1^e x \frac{1}{x} dx = \int_1^e dx = \underline{e-1}$

2)









3) $\frac{dE}{dt} = -\frac{dW}{dt}$

$E = \frac{1}{2} m \dot{x}^2, \quad \frac{dE}{dt} = m\dot{x}\ddot{x}$

$\frac{dW}{dt} = F\dot{x} = kx\dot{x}$

$\Rightarrow m\dot{x}\ddot{x} = -kx\dot{x}$

$\ddot{x} + \frac{k}{m}x = 0$

with $x(0) = x_0$

$\dot{x}(0) = v_0$

or similar but

$x_0 \neq 0$ or $v_0 \neq 0$

4) $1 + r_{eff} = \left(1 + \frac{r}{4}\right)^4 = \left(1 + \frac{r}{2} + \frac{r^2}{16}\right)^2 =$

$$= \left(1 + \frac{r}{2} + \frac{r^2}{16} + \frac{r}{2} + \frac{r^2}{4} + \frac{r^3}{32} + \frac{r^2}{16} + \frac{r^3}{32} + \frac{r^4}{16^2}\right)$$

$$= \left(1 + r + \frac{3}{8}r^2 + \frac{1}{16}r^3 + \frac{1}{256}r^4\right)$$

Thus:

$$r_{eff} - r = \frac{3}{8}r^2 + \frac{1}{16}r^3 + \frac{1}{256}r^4$$

5) $P'(t) - \delta P(t) = -x$ Use the integrating factor $e^{-\delta t}$

$$[P'(t) - \delta P(t)]e^{-\delta t} = \frac{d}{dt}\left[P(t)e^{-\delta t}\right]$$

$$\int_0^t \frac{d}{d\tau}\left[P(\tau)e^{-\delta\tau}\right]d\tau = P(t)e^{-\delta t}\Big|_0^t = P(t)e^{-\delta t} - P_0 =$$

$$= -\int_0^t x e^{-\delta\tau}d\tau = \frac{x}{\delta}\left[e^{-\delta\tau}\right]_0^t = \frac{x}{\delta}(e^{-\delta t} - 1)$$

Thus:

$$P(t)e^{-\delta t} - P_0 = \frac{x}{\delta}(e^{-\delta t} - 1)$$

$$P(t) = P_0 e^{\delta t} + \frac{x}{\delta}(1 - e^{\delta t})$$

$$= P_0 e^{\delta t} - \frac{x}{\delta}(e^{\delta t} - 1)$$