TEST #2

9:30 — 10:20am, November 17, 2006 DR. B. PROTAS

LAST (FAMILY) NAME: ______________________

FIRST (GIVEN) NAME: ______________________

STUDENT NUMBER: ______________________

STUDENT MARK: ______________________ (Max 10 points)

- The test has 3 questions, worth 3, 3, and 4 points; please provide your answers on the reverse side; you may also attach as many additional sheets as needed, but make sure to label them clearly;
- Time allowed: **50 minutes**
- Only the McMaster Standard Calculator Casio FX991MS is allowed

---

1. Imagine you have a container with volume V filled with water. Water is flowing out through a hole at the bottom of the container with velocity u. Assume that the gravitational acceleration is g. Using the principles of the dimensional analysis and the Buckingham Π Theorem determine the functional dependence of u on V and g, i.e., the form of the function $u = f(V,g)$.
(3 points)

2. You are given a second–order ordinary differential equation known as the *Duffing equation*
$$\ddot{x} + \omega_0^2 x + \beta x^3 = 0, \qquad (1)$$
where $\omega, \beta \in \mathbb{R}$ are constants and $\beta < 0$. Consider the equilibrium solution $x(t) = \dot{x}(t) = 0$ and:
   (a) characterize the type of this equilibrium and determine whether or not it is stable; is this equilibrium structurally stable?
   (b) find other equilibrium solutions of equation (1); how will these additional equilibria change if the sign of β is reversed?

   (3 points)

3. A beam is mounted so that its ends are at $x = 0$ and $x = 1$. There is a uniform (i.e., independent of x) load $h \geq 0$ applied to the beam. The left endpoint is fixed and there is a torque h^2 applied at the right endpoint. Consequently, the *deflection* $y(x)$ of the beam satisfies the following differential equation
$$\begin{cases} \dfrac{d^2y}{dx^2} = h, \quad x \in (0,1), \\ y(0) = 0, \quad \dfrac{dy}{dx}(1) = h^2. \end{cases} \qquad (2)$$
Determine the value h_{max} of the load h resulting in the *largest* deflection of the beam.
HINT — First find solution of equation (2), then determine its point of maximum deflection and eventually differentiate it with respect to h.
(4 points)