HOMEWORK #1

Due: January 29 (Tuesday) by midnight

Instructions:

- The assignment consists of *four* questions, worth 4, 2, 2 and 2 points.
- Submit your assignment *electronically* (via Email) to the address `math2t03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH2T03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. Design a MATLAB function that will create a $[4\times 4]$ matrix with $\mathbf{A}$ and a 4×1 *column* vector $\mathbf{x}$ with entries defined by the following expressions

$$\mathbf{A}_{i,j} = \begin{cases} ij(q^2+1), & \text{if } i \geq j, \\ ij(q-1), & \text{otherwise,} \end{cases} \qquad i,j = 1,\cdots,4$$

$$\mathbf{x}_i = \sum_{k=1}^{i} q^k, \qquad i = 1,\ldots,4$$

where q should be read from the keyboard. Then write a code that will perform the following operations:

(a) print out the entries of the matrix $\mathbf{A}$ and the vector $\mathbf{x}$,

(b) compute and print the value of the expression $\mathbf{x}^T\mathbf{A}\mathbf{x}$,

(c) compute and print out the *dyadic* product of the vector $\mathbf{x}$ with itself,

(d) using the following three vectors $\mathbf{a} = [1\ \ 2\ \ 3]^T$, $\mathbf{b} = [2\ \ -4\ \ 6]^T$ and $\mathbf{c} = [1\ \ 3\ \ 5]^T$ illustrate the vector identity (i.e., show that the left–hand side = right–hand side)

$$\mathbf{c}\times(\mathbf{a}\times\mathbf{b}) = (\mathbf{ab}-\mathbf{ba})\mathbf{c},$$

where $\mathbf{ab}$ denotes the dyadic product of the vectors $\mathbf{a}$ and $\mathbf{b}$.

HINT — the code of your functions should be appended at the end of the submission template, rather than submitted as separate files.
(4 points)

2. Your are given the following three real–valued functions:

$$y = f(x) = x^3, \qquad x \in \Omega_1 = [x_1, x_2],$$

$$y = g(x) = \ln(1+x^2), \qquad x \in \Omega_2 = \left[\frac{x_1}{2}, \frac{3x_2}{2}\right],$$

$$y = h(x) = \arctan(x), \qquad x \in \Omega_3 = \left[\frac{x_1}{4}, \frac{5x_2}{4}\right],$$

where $x_1, x_2 > 0$ ($x_1 < x_2$) are parameters that should be entered by the user from the keyboard. Write a MATLAB code that will plot these three functions on two separate figures

(a) for $x \in \Omega_{\cup} = \bigcup_{i=1}^{3} \Omega_i$ (the "union" domain),

(b) for $x \in \Omega_{\cap} = \bigcap_{i=1}^{3} \Omega_i$ (the "intersection" domain),

For every function use a different color and line type. In each figure use 100 grid points to plot the functions and also generate appropriate axis labels and legends.
(2 points)

3. You are given a complex–valued function of the complex variable $f : \mathbb{C} \to \mathbb{C}$, where $f(z) = \ln(z)$ and $z = x + iy$ ($i = \sqrt{-1}$). Assuming that $-1 \leq x, y \leq 1$, plot the surface $\Im(f(z)) = F(x,y)$ as a function of x and y. Use a step size of your choice.
(2 points)

4. Write a MATLAB code which will estimate the level of round–off errors in a computer by performing the following operations:

   (a) without using the intrinsic function `eps`, determine the value of ε for which $1.0 + \varepsilon$ becomes indistinguishable from 1.0,

   (b) display where the round–off error appears by drawing a log–linear plot of $\frac{(1.0+\varepsilon)-1.0}{\varepsilon}$ as a function of ε.

   HINT — In order to generate a log–linear plot use the command `semilogx`.
   (2 points)