HOMEWORK #5

Due: April 8 (Tuesday) by midnight

Instructions:

- The assignment consists of *three* questions, worth 2, 4, and 4 points.
- Submit your assignment *electronically* (via Email) to the address `math2t03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH2T03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. Use one of the versions of the QR factorization to construct an operator representing the projection onto range$(\mathbb{A})^{\perp}$, where the matrix $\mathbb{A}$ is given as

$$\mathbb{A} = \begin{bmatrix} 1 & 5 & 0 \\ 3 & 5 & 8 \\ 0 & 9 & 4 \\ 3 & 3 & 2 \\ 2 & 6 & 7 \end{bmatrix}.$$

   Employ this operator to determine the projection of the vector $\mathbf{a} = [0\,0\,1\,0\,0]^T$ onto range$(\mathbb{A})^{\perp}$. *[2 points]*

2. Consider the following system of linear equations

$$\begin{aligned} x &= -1, \\ 3x &= 2. \end{aligned} \tag{1}$$

   Using the MATLAB codes discussed during the lectures

   (a) compute the $\mathbb{Q}$ and $\mathbb{R}$ matrices obtained via the reduced and full QR factorizations of the matrix $\mathbb{A}$ representing system (1),

   (b) determine the left inverse of the matrix $\mathbb{A}$,

   (c) find the least–squares solution x^* of system (1),

   (d) plot on a single figure:

      i. range$(\mathbb{A})$ (using blue dotted line),

      ii. $\mathbb{A}x^*$ (using a blue star),

      iii. b (using a red circle),

      iv. $(b - \mathbb{A}x^*)$ (using a red dashed line).

      The figure should correspond to the range $[-2, 2]$ for Y and should use the same scale for the X and Y coordinates (you can obtain this with the MATLAB command `axis equal`).

   [4 points]

3. You are given a set of points $\{x_i, y_i\}_{i=1}^{5}$, where

$$x_i \in \{77\ 100\ 185\ 239\ 285\},$$
$$y_i \in \{2.4\ 3.4\ 7.0\ 11.1\ 19.6\}.$$

Find (and print out) the values of the parameters α and β which minimize the least–squares errors $E(\alpha,\beta) = \sum_{i=1}^{5}[y_i - f(x_i)]^2$ for the following approximating functions

(a) $f(x) = \alpha e^{\beta x}$,

(b) $f(x) = \alpha x^{\beta}$.

In each case also determine the corresponding least–squares errors, i.e., the residuals $E(\alpha,\beta)$ corresponding to the optimal choices of the parameters α and β. Produce a plot that will show the original data points (marked with stars) together with the approximating functions (a) and (b) with the optimal parameters α and β (marked as lines of different types).
HINT — a nonlinear change of variables is required to bring interpolating functions (a) and (b) to a form consistent with the linear regression.
[4 points]