HOMEWORK #2: EULER's METHOD FOR INITIAL–VALUE PROBLEMS

Due: one minute after 11:59pm on October 13

Instructions:

- The assignment consists of *four* questions worth, respectively, 2, 2, 3, and 3 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2Z03a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), section 9.1.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 2.6, 6.1.

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1. Compute the value $y(2)$ as the solution of the following initial–value problem (IVP)

$$y' = \frac{1}{1+t^2} - 2y^2, \qquad y(0) = 0 \tag{1}$$

   using *Euler's explicit method* with the time steps $h = 0.1$ and $h = 0.01$. Save the results in the variables `Answer1` and `Answer2`, respectively.

2. Solve IVP (1) analytically. Using the MATLAB function `plot` show the exact y_{ex} and numerical y_{num} solutions obtained in Problem 1 with the time step $h = 0.1$ for $t \in [0,2]$ on the same graph. Use different line types and colours to distinguish these two solutions. The graph should appear as `Figure 1`.

3. Compute the error of the numerical approximation at $t = 1$ for IVP (1) corresponding to different time steps $h = 2^{-(n+1)}$, $n = 1,\ldots,12$. Plot this error using the MATLAB function `loglog` as a function of h and draw conclusion about the accuracy of Euler's method. The graph should appear as `Figure 2`, whereas your conclusion should be saved in `Answer3` (as text).
   (**Hint:** the error of the numerical method is defined as the distance $|y_{num}(1) - y_{ex}(1)|$).

4. Obtain the numerical solution of IVP (1) using *explicit Heun's method* defined by the formula

$$y_{n+1} = y_n + h\frac{f(t_n,y_n) + f(t_{n+1},y^*_{n+1})}{2},$$

where

$$y^*_{n+1} = y_n + hf(t_n,y_n).$$

Similarly to the Problem 3, compute the error at $t = 1$ corresponding to different time steps $h = 2^{-(n+1)}$, $n = 1,\ldots,12$. Plot this error using the `loglog` function as a function of h and draw conclusion about the accuracy of Heun's method. The graph should appear as `Figure 3`, whereas your conclusion should be saved in `Answer4` (as text).