HOMEWORK #3: METHODS FOR ODEs: EULER's AND HEUN's METHODS AND APPLICATIONS TO HIGHER-ORDER IVPs

Due: one minute after 11:59pm on October 26

Instructions:

- The assignment consists of *four* questions worth, respectively, 2, 2, 2, and 4 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2Z03a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), section 9.2.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 3.7, 6.1, 6.4.

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1. Consider the following initial-value problem (IVP):

$$y' = -\lambda y + t, \qquad y(0) = 0, \tag{1}$$

with $\lambda = 1$ and $\lambda = 10$. Apply *Euler's method*

$$y_{n+1} = y_n + h f(t_n, y_n) \tag{2}$$

to approximate the solution of (1) for $t \in [0,3]$ with the time step $h = 0.1$. Plot the numerical solutions for both values of λ on the same graph using different line colours. The graph should appear as `Figure 1`.

2. Approximate the solution of the following IVP

$$y' = (3t^2 - 2t + 1)y, \qquad y(0) = 1 \tag{3}$$

using *Heun's method*

$$y_{n+1} = y_n + \frac{1}{2} h \left[f(t_n, y_n) + f(t_{n+1}, y_{n+1}) \right] \tag{4}$$

for $t \in [0,2]$ with the time steps $h = 0.1$ and $h = 0.05$. Plot the distance between the exact and numerical solutions versus t using the MATLAB function `semilogy` and different line colours. The graph should appear as `Figure 2`.

3. Approximate the solution of IVP (3) with *modified Euler's method*

$$y_{n+1} = y_n + h\, f(t_{n+\frac{1}{2}}, y_{n+\frac{1}{2}}) \tag{5}$$

for $t \in [0,2]$ with the time steps $h = 0.1$ and $h = 0.05$. Plot the distance between the exact and numerical solutions versus t using the MATLAB function `semilogy` and different line colours. The graph should appear as `Figure 3`.

4. Consider the following IVP

$$x^2y'' - 2xy' + 2y = 0, \qquad y(1) = 4, \qquad y'(1) = 9. \tag{6}$$

Transform this equation to a system of first–order differential equations which should then be solved using *Euler's method* (2), *Heun's method* (4) and *modified Euler's method* (5) for $x \in [1,2]$ and with $h = 0.1$. Find the analytical solution for problem (6) and save the difference $\Delta = |y_{num}(2) - y_{ex}(2)|$ in `Answer1` in the format $[\Delta_{Eul} \ \ \Delta_{Heun} \ \ \Delta_{EulMod}]$. Plot also the exact and all three numerical solutions on the same graph using different line colours. The graph should appear as `Figure 4`.
(**Hint:** Given the system system of ODEs

$$\begin{cases} u' = f(t,u,v), \\ v' = g(t,u,v), \end{cases}$$

with initial values $u(t_0) = u_0$, $v(t_0) = v_0$, its numerical solution using Euler's method can be obtained as follows:

$$\begin{aligned} u_{n+1} &= u_n + h\, f(t_n, u_n, v_n), \\ v_{n+1} &= v_n + h\, g(t_n, u_n, v_n). \end{aligned}$$

Heun's method and modified Euler's method are implemented in an analogous way.)