HOMEWORK #4: HIGHER ORDER ODEs: SPRING – MASS SYSTEM

Due: one minute after 11:59pm on November 9

Instructions:

- The assignment consists of *four* questions worth, respectively, 2, 3, 3, and 2 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2Z03a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), section 9.2.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 3.8.1–3.8.3.

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1. Consider a *spring–mass system without forcing and dumping* described by the following initial–value problem (IVP)

$$x'' + x = 0, \qquad x(0) = 1, \qquad x'(0) = -1. \tag{1}$$

   Solve IVP (1) analytically. Using the MATLAB function `plot` show the solution $x(t)$ for $t \in [0, 10]$ with the time step size $h = 0.2$. The graph should appear as `Figure 1`.

2. Approximate the solution of IVP (1) with *modified Euler's method*. Plot the *phase portrait* $(x(t), x'(t))$ for $t \in [0, 10]$ for three different initial values

$$(x_0, x_0')_1 = (1, 2), \qquad (x_0, x_0')_2 = (-3, 5), \qquad (x_0, x_0')_3 = (2, -3)$$

   and the time step size $h = 0.2$. The graph should appear as `Figure 2` and should use different line colours for different solutions.

3. Consider a *damped spring–mass system without forcing* described by the IVP

$$x'' + \alpha x' + x = 0, \qquad x(0) = 1, \qquad x'(0) = 1. \tag{2}$$

   Approximate the solution of IVP (2) with *Heun's method*. Plot the phase portraits $(x(t), x'(t))$ for $t \in [0, 20]$ with the time step size $h = 0.2$ for $\alpha = 0.1$ and $\alpha = 0.5$ on the same graph using different line colours. The graph should appear as `Figure 3`.

4. Consider a *nonlinear spring–mass system* described by the IVP

$$x'' + \sin x = 0, \qquad x(0) = -1, \qquad x'(0) = 1. \tag{3}$$

Approximate the solution of IVP (3) with Euler's and Heun's methods. Plot the phase portraits $(x(t), x'(t))$ for $t \in [0, 10]$ with the time step size $h = 0.1$ on the same graph using different line colours. The graph should appear as `Figure 4`.