HOMEWORK #5: LINEAR ALGEBRA: EIGENVALUE PROBLEMS

Due: one minute after 11:59pm on November 23

Instructions:

- The assignment consists of *four* questions worth, respectively, 2, 2, 3, and 3 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2Z03a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), sections 4.1–4.4.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 8.8–8.12.

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1. Find the eigenvalues and eigenvectors of the following matrix $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 6 & 1 & -3 & 3 \\ 0 & 3 & 2 & -1 \\ 1 & 9 & -4 & 1 \\ 0 & -3 & 2 & 7 \end{bmatrix}.$$

   Save the eigenvalue with the largest magnitude in the variable `Answer1` and the corresponding eigenvector in the variable `Answer2`. Also, find the eigenvalues and eigenvectors of the transposed matrix $\mathbf{A}^T$ and save the eigenvalue with the largest magnitude in the variable `Answer3` and the corresponding eigenvector in the variable `Answer4`.

2. Find the matrix $\mathbf{P}$ that diagonalizes matrix $\mathbf{A}$ from Problem 1 and the diagonal matrix $\mathbf{D}$ such that $\mathbf{D} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}$. Save the matrices $\mathbf{P}$ and $\mathbf{D}$ in the variables `Answer5` and `Answer6`, respectively.

3. Consider the matrix $\mathbf{B}$

$$\mathbf{B}(s) = \begin{bmatrix} 4 & 5-5s \\ 2.5+5s & -1 \end{bmatrix}$$

parameterized with the real variable s. Find the eigenvectors of the matrix $\mathbf{B}$ for $s \in [-1,1]$ with the step size $h = 0.01$ and plot the absolute value of the cosine of the angle between the eigenvectors $\mathbf{v}_1(s)$ and $\mathbf{v}_2(s)$

$$\cos\left(\widehat{\mathbf{v}_1,\mathbf{v}_2}\right) = \frac{\mathbf{v}_1 \cdot \mathbf{v}_2}{|\mathbf{v}_1||\mathbf{v}_2|}$$

as a function of the parameter s. The graph should appear as `Figure1`. Find the value of s for which $\mathbf{v}_1$ and $\mathbf{v}_2$ are orthogonal to each other and save this value in the variable `Answer7`.

4. A nonzero $n \times n$ matrix $\mathbf{A}$ is said to be *nilpotent of index m* if m is the smallest positive integer for which $\mathbf{A}^m = 0$. Write a MATLAB code which will determine if a given matrix is nilpotent and will find its index. Perform your computations for the matrix

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \end{bmatrix}$$

and save the result, i.e., the index m, in the variable `Answer8`.