HOMEWORK #6: HOMOGENEOUS SYSTEMS OF LINEAR ODEs

Due: one minute after 11:59pm on December 7

Instructions:

- The assignment consists of *four* questions worth, respectively, 3, 2, 2, and 3 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where “`Name`” is your last name, “`XXXXXX`” is your student ID number, and “`N`” is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2Z03a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. ”**Numerical Mathematics**” by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), sections 1.6, 9.2.
  2. ”**Advanced Engineering Mathematics**” by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), section 10.2.

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1. Consider the system of linear ODEs

$$\begin{cases} x' = -\frac{5}{2}x + 2y, \\ y' = \frac{3}{4}x - 2y \end{cases} \tag{1}$$

which can be written in the form $\mathbf{x}' = \mathbf{A}\mathbf{x}$. Find all eigenvalues of the matrix $\mathbf{A}$ of system (1) and save them in the variable `Answer1` in the format $[\lambda_1 \ \ \lambda_2]$. Also, find all eigenvectors of $\mathbf{A}$ and obtain equations for the lines in the direction of these eigenvectors in the Cartesian coordinate system. Solve system (1) for $t \in [0,2]$ assuming the following four different initial conditions

$$(x_0,y_0)_1 = (-20,20), \qquad (x_0,y_0)_2 = (-25,5),$$
$$(x_0,y_0)_3 = (10,-20), \qquad (x_0,y_0)_4 = (20,0)$$

and using Euler's method with $h = 0.1$. Then plot the solutions $\{x_i(t), y_i(t)\}, i = 1,2,3,4$ in the (x,y) coordinates. Include also the analytical solutions of system (1) for all initial conditions and the lines representing the directions of the eigenvectors (use different line types and colours). The graph should appear as `Figure1`.

2. Apply *Heun's method* to system (1) and obtain numerical solutions for $t \in [0,2]$ using the initial values

$$(x_0, y_0)_5 = (5, 15), \qquad (x_0, y_0)_6 = (10, -10)$$

and the time step size $h = 0.1$. Plot both solutions using the (x, y) coordinates in `Figure2` and include also the same elements as shown in `Figure1` (the corresponding analytical solutions and the lines representing the direction of the eigenvectors).

3. Consider the following system of linear ODEs

$$\begin{cases} x' = 2x + y + 2z, \\ y' = 3x + 6z, \\ z' = -4x - 3z. \end{cases} \tag{2}$$

Solve system (2) analytically using the following two sets of initial conditions

$$x(0) = 10, \qquad y(0) = 20, \qquad z(0) = -20,$$

and

$$x(0) = 10, \qquad y(0) = 20, \qquad z(0) = 10.$$

Plot the two trajectories in $\mathbb{R}^3$ for $t \in [0,4]$ with the time step $h = 0.1$ and using different line colours. The graph should appear as `Figure3`.
(**Hint:** Use the MATLAB function `plot3` to plot 3–D curves).

4. Apply *Euler's method* to system (2) and obtain a numerical solution corresponding to the initial values

$$x(0) = -10, \qquad y(0) = 20, \qquad z(0) = -20$$

for $t \in [0,2]$ with the time step size $h = 0.05$. Plot the numerical solution together with the corresponding analytical solution as a function of time t with the x components appearing in `Figure4`, y components appearing in `Figure5` and z components appearing in `Figure6`. Use different line types and colours for the analytical and numerical solutions.