TEST #1 (VERSION 1)

19:00 — 20:15, October 6, 2009

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- This text paper consists of 6 pages (including this one). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator.
- There are 16 multiple–choice questions worth 1 mark each (no part marks).
- The questions must be answered on the COMPUTER CARD with an HB PENCIL.
- Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing).
- Only the McMaster Standard Calculator Casio FX991MS is allowed

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Computer Card Instructions

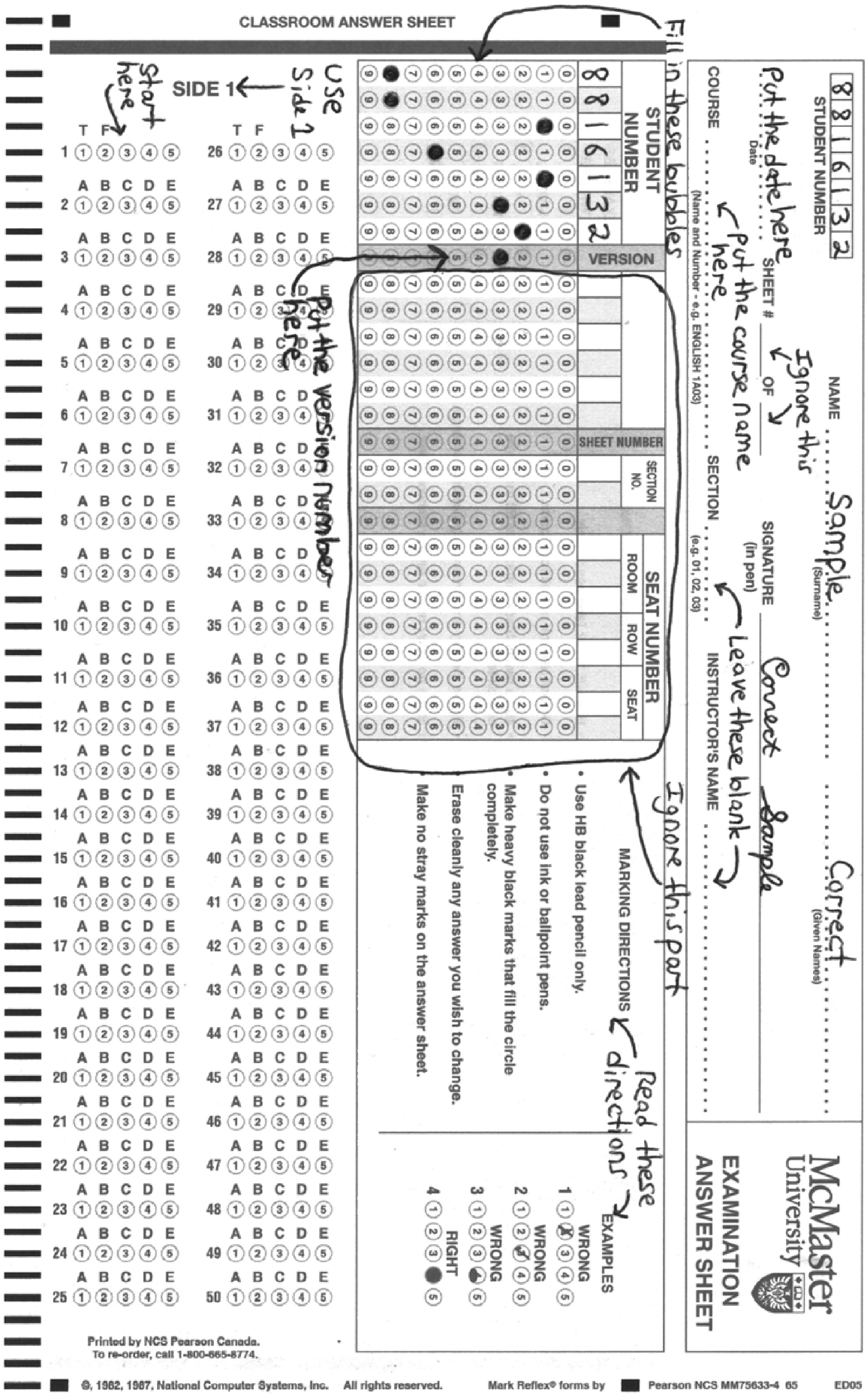
NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will NOT be sensed. Erasures must be thorough or the scanner may still sense a mark. Do NOT use correction fluid on the sheets. Do NOT put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet MUST be signed in the space marked SIGNATURE.
2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only ONE choice from the alternatives (A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked “1”.

See Figure on the next page for additional information on filling computer cards.

Sample of how to fill out the computer card:



1. We define in MATLAB the following two vectors:

```
>>  a=[7;2;-9]; b=[0;-1;3];
```

The inner (scalar) product of the two vectors can be computed with the following command:

(a) `a*b`

(b) `a*b'`

(c) `a'*b`

(d) `prod(a,b)`

(e) none of the above

2. Which of the following statements is true:

(a) the equation $y(y-1)dx-(x+1)dy=0$ is linear in y, but not linear in x,

(b) the equation $y(y-1)dx-(x+1)dy=0$ is linear in x, but not linear in y,

(c) the equation $y(y-1)dx-(x+1)dy=0$ is linear both in x and in y,

(d) the equation $\frac{dy}{dx}=|x|$ is not linear in y

(e) none of the above

3. The function $y(x)=\frac{1}{\cos(x)+C}$, where C is a constant, satisfies the differential equation $\frac{dy}{dx}=\sin(x)y^2$. The solution of the initial value problem corresponding to the initial condition $y(0)=2/3$ is defined on the interval

(a) $0\leq x<\frac{2\pi}{3}$

(b) $0\leq x\leq\frac{2\pi}{3}$

(c) $0\leq x<\frac{\pi}{3}$

(d) $0\leq x\leq\frac{\pi}{3}$

(e) $0\leq x<\infty$

4. You are given the equation $\frac{dy}{dt}=y(1-y)$. Assuming that C is a real number, determine which statement is true:

(a) all solutions of the equation have the form $y(t)=\frac{1}{1+Ce^{t}}$,

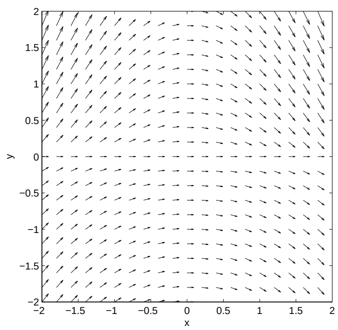
(b) all *non–constant* solutions of the equation have the form $y(t)=\frac{1}{1+Ce^{t}}$,

(c) all solutions of the equation have the form $y(t)=\frac{1}{1+Ce^{-t}}$,

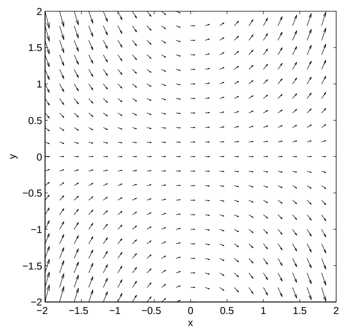
(d) all *non–constant* solutions of the equation have the form $y(t)=\frac{1}{1+Ce^{-t}}$,

(e) none of the above

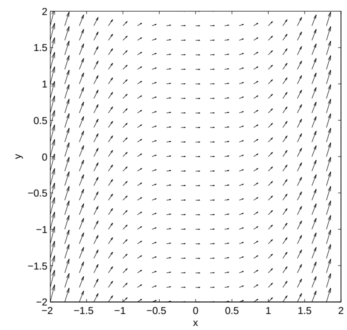
5. Which of the following directions fields corresponds to the equation $\frac{dy}{dx} = xy$?



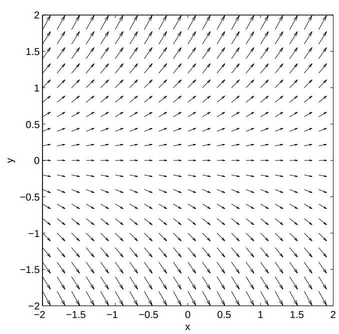
(a)



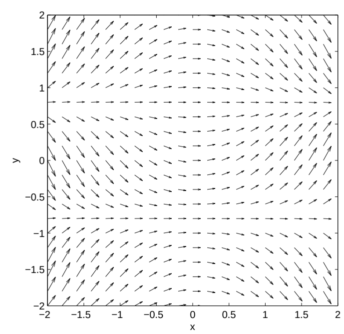
(b)



(c)



(d)



(e)

6. Let $y(x)$ be the unique solution of the initial value problem $\frac{dy}{dx} = e^{\frac{1}{y}}$, $y(0) = 2$. Then, the second derivative of $y(x)$ at the point $x = 0$ is (HINT — do not attempt to solve the equation!)

(a) $\frac{e}{2}$

(b) $-e$

(c) $\frac{e^2}{4}$

(d) 0

(e) $-\frac{e}{4}$

7. The solution of the equation $xy' - y = x^3$ is

(a) $y = \frac{x^4}{3} + cx$

(b) $y = \frac{x^3}{4} - \frac{c}{x}$

(c) $y = \frac{x^3}{2} + cx$

(d) $y = x^3 + cx$

(e) $y = \frac{x^2}{2} + c$

(f) None of the above

8. The solution of the initial value problem $(1+\cos(x))y' - (\sin(x))y = 2x$, $y(0) = 1$ is

   (a) $y = -2x\cot(x) + \frac{1}{\sin(x)} + 2$
   (b) $y = \frac{x^2+2}{1+\cos(x)}$
   (c) $y = \frac{x^2+1}{1+\cos(x)}$
   (d) $y = 2x - \tan(x) + 1$
   (e) $y = \frac{x^2+2}{1+\sin(x)}$
   (f) None of the above

9. The number of bacteria in a liquid culture is observed to grow at a rate proportional to the number of cells present. At the beginning of the experiment there are 10,000 cells and after three hours there are 500,000. How many will there be approximately after one day of growth if this unlimited growth continues?

   (a) 3.34×10^9
   (b) 3.9×10^{17}
   (c) 3.76×10^{24}
   (d) 4×10^{33}
   (e) None of the above

10. Determine the solution of the initial value problem $\frac{dP}{dt} = 0.08P\left(1 - \frac{P}{1000}\right)$, $P(0) = 100$. The approximate time required for the population to reach 900 is

    (a) 1012.5
    (b) 6.48
    (c) 113.79
    (d) 54.9
    (e) None of the above

11. Which of the following sets of functions is linearly dependent:

    (a) $\{2+x, 2-x, x^2\}$
    (b) $\{e^{2+x}, e^{2-x}, e^2\}$
    (c) $\{e^{x+2}, e^{x-2}, x^2\}$
    (d) $\{1, \cos(x), \sin(x)\}$
    (e) $\{x, \sqrt{(}x), x\sqrt{(}x)\}$

12. Consider the initial-value problem $(3+x)y''(x) + x^2y'(x) - (1-x)^{\frac{1}{2}}y(x) = x$, $y(0) = 1$, $y'(0) = -1$. In which of the following intervals does the problem have a unique solution:

    (a) $(-3, 3)$
    (b) $(-3, 1)$
    (c) $(-\infty, 1)$
    (d) $(1, 3)$
    (e) $(-\infty, \infty)$

13. The homogeneous differential equation $y'' - y' - 6y = 0$ has a fundamental set of solutions

    (a) $\{e^{3x}, e^{-2x}\}$

    (b) $\{e^{-3x}, e^{2x}\}$

    (c) $\{e^{-x}, e^{-6x}\}$

    (d) $\{x^3, x^{-2}\}$

    (e) $\{e^{x/3}, e^{-x/2}\}$

14. The Wronskian determinant of the set of functions $\{1, e^x, xe^x\}$ is

    (a) 1

    (b) e^x

    (c) xe^x

    (d) e^{2x}

    (e) xe^{2x}

15. The homogeneous equation $y'' + 4y = 0$ has a general solution $y = C_1 \cos(2x) + C_2 \sin(2x)$ (you need not check that). The boundary–value problem $y'' + 4y = 0$, $y(0) = 0$, $y(\pi) = 1$ has:

    (a) unique solution $y = 1 - \cos(2x)$

    (b) no solution

    (c) unique solution $y = \sin(2x)$

    (d) two solutions $y = \cos(2x)$ and $y = \sin(2x)$

    (e) infinitely many solutions $y = C \sin(2x)$, where C is any number

16. The homogeneous differential equation $xy'' - 2y' = 0$, $(x > 0)$ has a general solution $y = C_1 + C_2 x^3$ (you need not check that). For this equation, the initial value problem $y(1) = 0$, $y'(1) = 1$ has solution

    (a) $y = 0$

    (b) $y = x^3$

    (c) $y = 1 - x^3$

    (d) $y = -\frac{1}{3} + \frac{1}{3}x^3$

    (e) $y = \frac{2}{3} + \frac{1}{3}x^3$