TEST #2 (VERSION 3)

19:00 — 20:15, November 10, 2009

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- This text paper consists of 8 pages (including this one). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator.
- There are 16 multiple–choice questions worth 1 mark each (no part marks).
- The questions must be answered on the COMPUTER CARD with an HB PENCIL.
- Make sure to indicate the **test version** and your **student number**.
- Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing).
- Only the McMaster Standard Calculator Casio FX991MS is allowed

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Computer Card Instructions

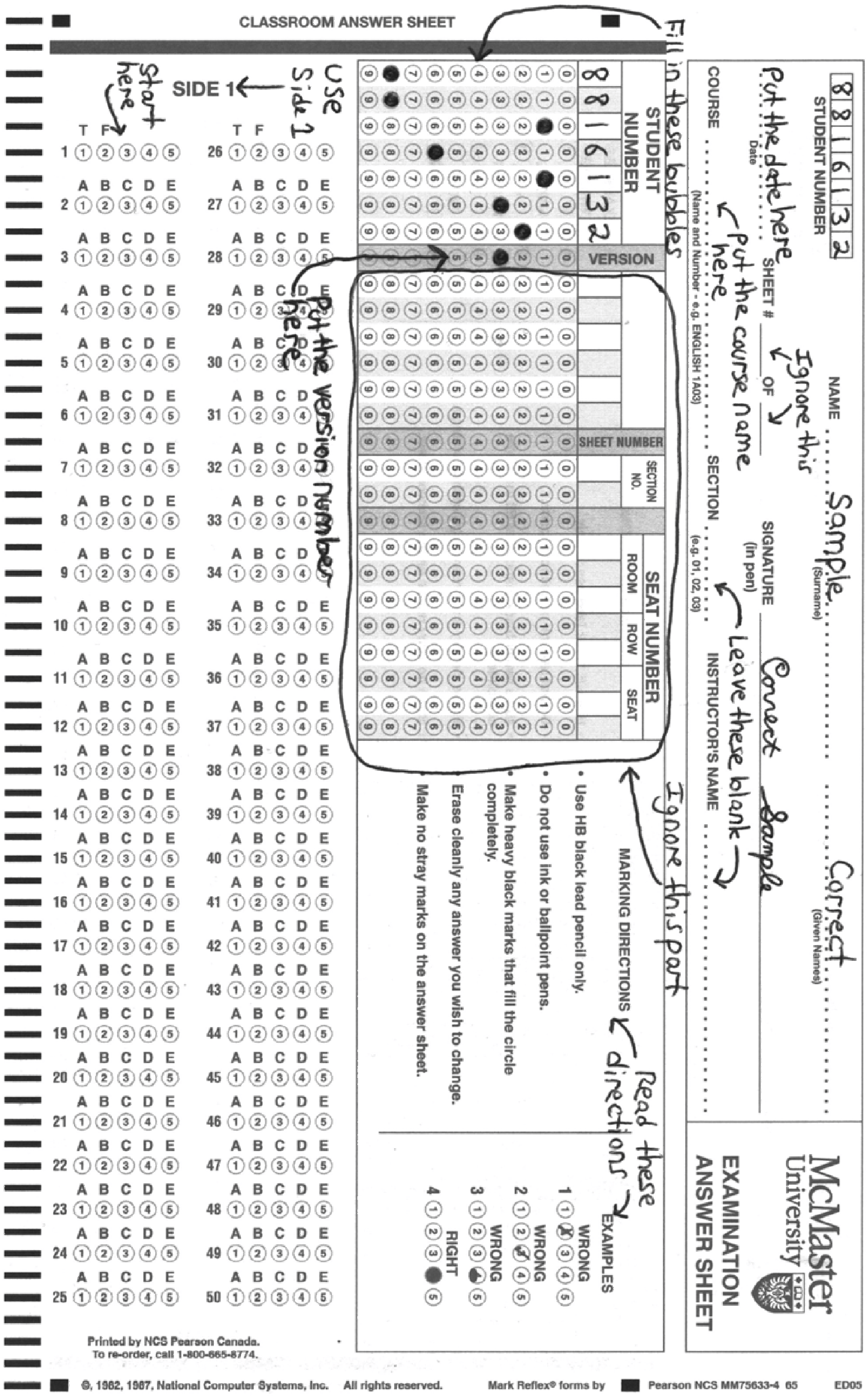
NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will NOT be sensed. Erasures must be thorough or the scanner may still sense a mark. Do NOT use correction fluid on the sheets. Do NOT put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet MUST be signed in the space marked SIGNATURE.
2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only ONE choice from the alternatives (A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked “1”.

See Figure on the next page for additional information on filling computer cards.

Sample of how to fill out the computer card:



1. The general solution of the differential equation $x^2y'' - 3xy' + 4y = x$ for which the homogeneous part has the solutions $y_1 = x^2$, $y_2 = x^2\ln(x)$ is

   (a) $y(x) = c_1x^2 + c_2x^2\ln(x) - x^3 + x^3\ln(x)$

   (b) Solution does not exist

   (c) $y(x) = c_1x^2 + c_2x^2\ln(x) + x$

   (d) $y(x) = c_1x^2 + c_2x^2\ln(x) - 2x\ln(x) + x$

   (e) $y(x) = c_1x^2 + c_2x^2\ln(x) - x$

2. A spring with a mass of 2 kg has natural length of 0.7 m. A force of 25.6 N is required to stretch the length of 0.9 m. If the spring is stretched to a length of 0.9 m and then released with an initial velocity 0, find the position of the mass at any time t

   (a) $x(t) = \frac{1}{5}\cos(8t)$

   (b) $x(t) = \frac{1}{2}\cos(8t)$

   (c) $x(t) = \frac{10}{7}\cos(8t)$

   (d) $x(t) = \frac{1}{10}e^{8t} + \frac{1}{10}e^{-8t}$

   (e) $x(t) = \frac{1}{10}\cos(8t) + \frac{1}{10}\sin(8t)$

3. The deflection $y(x)$ $(0 \le x \le 1)$ of a beam embedded at its right end and simply supported at its left end satisfies the boundary–value problem

$$\begin{aligned} &y^{(4)}(x) = 48, \\ &y(0) = 0, \;\; y''(0) = 0, \\ &y(1) = 0, \;\; y'(1) = 0. \end{aligned}$$

   The shape of the deflection curve is given by

   (a) $y(x) = -x + 3x^3 - 2x^4$

   (b) $y(x) = x^2(1-x)^2$

   (c) $y(x) = x - 3x^3 + 2x^4$

   (d) $y(x) = 2x^4$

   (e) $y(x) = 2x^2 - 2x^4$

4. The solution of the boundary–value problem $y'' + y = 1$, $y(0) = 0$, $y(\pi) = 0$ is

   (a) $y(x) = 1 - \cos(x)$

   (b) $y(x) = 1 - \cos(x) + C\sin(x)$, C arbitrary

   (c) $y(x) = 1 + \sin(x)$

   (d) $y(x) = \cos(x) - \sin(x) - 1$

   (e) none of the above

5. Consider the boundary–value problem $y'' + \lambda y = 0$, $y'(0) = 0$, $y(\pi) = 0$. The smallest eigenvalue λ of this problem is

   (a) 0

   (b) $\frac{\pi}{2}$

   (c) $\frac{1}{2}$

   (d) $\frac{1}{4}$

   (e) π

6. The inverse matrix to $A = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$ is

   (a) $A^{-1} = \begin{bmatrix} -1 & -1 \\ -3 & -2 \end{bmatrix}$

   (b) $A^{-1} = \begin{bmatrix} 1 & 1 \\ 1/3 & 1/2 \end{bmatrix}$

   (c) $A^{-1} = \begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix}$

   (d) $A^{-1} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$

   (e) non–existent, because A is singular

7. The vector space of polynomials of degree 3 or less has dimension

   (a) 5

   (b) 4

   (c) 3

   (d) 2

   (e) 1

8. The rank of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ 1 & 5 & 3 \\ 1 & 1 & 1 \end{bmatrix}$ is

   (a) 3

   (b) 2

   (c) 1

   (d) 0

   (e) not defined

9. Given the vector `a=[1 2 3 4 5]`, the vector `[1 8 27 64 125]` whose entries are cubes of the entries of the vector `a` can be computed in MATLAB as follows

   (a) `a^3`
   (b) `a*a*a`
   (c) `a.^3`
   (d) `cube(a)`
   (e) `a*a^2`

10. The function $y(x) = C_1 \cosh(2x) + C_2 \sinh(2x) + \sin(x)$, where C_1 and C_2 are constants, is a solution of the differential equation

    (a) $y'' - 4y = -\sin(x)$
    (b) $y'' + y = -5\sin(x)$
    (c) $y'' + 4y = -5\sin(x)$
    (d) $y'' - 4y = -5\sin(x)$
    (e) $y'' - y = -5\sin(x)$

11. The general solution (complementary function) of the equation $\frac{d^4y}{dx^4} - 8\frac{d^2y}{dx^2} + 16y = 0$ is

    (a) $y_c(x) = C_1e^{2x} + C_2xe^{2x} + C_3x^2e^{2x} + C_4x^3e^{2x}$
    (b) $y_c(x) = C_1e^{2x} + C_2xe^{2x} + C_3e^{-2x} + C_4xe^{-2x}$
    (c) $y_c(x) = C_1e^{-2x} + C_2xe^{-2x} + C_3x^2e^{-2x} + C_4x^3e^{-2x}$
    (d) $y_c(x) = C_1x + C_2x^2 + C_3x^3 + C_4x^4$
    (e) $y_c(x) = C_1x^2 + C_2x^2(\ln(x)) + C_3x^{-2} + C_4x^{-2}(\ln(x))$

    HINT — note that $m^4 - 8m^2 + 16 = (m-2)^2(m+2)^2$.

12. The equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 5y = 0$ has the following fundamental solution set

    (a) $\{e^{2x}, xe^{2x}\}$
    (b) $\{e^{2x}, e^{-2x}\}$
    (c) $\{\cos(x), \sin(x)\}$
    (d) $\{e^x\cos(2x), e^x\sin(2x)\}$
    (e) $\{e^{2x}\cos(x), e^{2x}\sin(x)\}$

13. The particular solution of the differential equation $y'' + 4y = -1 + x^2$ is

    (a) $y_p(x) = \frac{1}{4}x^2 - \frac{3}{8}$

    (b) $y_p(x) = \frac{1}{2}x - \frac{3}{4}$

    (c) $y_p(x) = -1 + x^2$

    (d) $y_p(x) = -x + \frac{1}{3}x^3$

    (e) $y_p(x) = \frac{1}{4}x^2 + \frac{3}{8}$

14. The particular solution of the differential equation $y'' - 2y' + y = xe^x$ has the form (A, B, C, D are constants)

    (a) $y_p(x) = Ae^x$

    (b) $y_p(x) = (Ax + B)e^x$

    (c) $y_p(x) = (Ax^2 + Bx + C)e^x$

    (d) $y_p(x) = (Ax^3 + Bx^2 + Cx + D)e^x$

    (e) none of the above

15. The general solution of the equation $y'' - 2y' + y = \frac{e^x}{1+x^2}$ is

    (a) none of the following

    (b) $y(x) = c_1e^x + c_2xe^x - e^x \ln(1 + 2x) + xe^x \arctan(x)$

    (c) $y(x) = c_1e^x + c_2xe^x - \frac{1}{2}e^x \ln(1 + x^2) + xe^x \arctan(x)$

    (d) $y = c_1e^x + c_2xe^x - \frac{1}{2}e^x \ln(1 + x^2) + xe^x \frac{1}{1+x^2}$

    (e) $y = c_1e^x + c_2e^{-x}$

16. Consider the differential equation $4t^2y''(t) - ty'(t) + y(t) = 0$. The fundamental solution set is given by

    (a) $\{e^{t/4}, e^t\}$

    (b) $\{t^{1/4}, t\}$

    (c) $\{e^{t/8}\cos(\frac{\sqrt{15}}{8}), e^{t/8}\sin(\frac{\sqrt{15}}{8})\}$

    (d) $\{t^{1/8}\cos\frac{(\sqrt{15}}{8}), t^{1/8}\sin\frac{(\sqrt{15}}{8}$

    (e) $\{t^{1/8}\cos\frac{(\sqrt{15}}{8}), t^{1/8}\sin\frac{(\sqrt{15}}{8})\}$

(rough work)

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