Math 2MM3: Final Exam

Day Class Instructors: Dr. Gordon Craig and Dr. Zdislav Kovarik

Wednesday, April 23, 2008

Duration of the examination: 3 hours

Name: (print) ____

Student No. : ____

Tutorial Section No. : ____

McMaster University Final Examination, April 2008

This examination paper includes 15 pages (including this title page) and 10 questions. The total sum of the marks is 100. It is your responsibility to ensure that your copy of this paper is complete. Please bring any discrepancy to the attention of your invigilator.

Marks: (do not enter answers here)

1	2	3	4	5	6
7	8	9	10		total

Special instructions: **READ CAREFULLY!**

- This exam is open book and open notes
- Write the answers to the questions in the space provided. If you need more space you may use the blank pages at the end of the exam paper.
- This paper must be returned with all pages included.

- Explain your steps. Your mark will depend on how clear and complete your solutions are.
- The textbook is allowed, but no notes are permitted.
- Only the standard McMaster calculator Casio fx 991 is allowed on this exam.
- You may evaluate integrals by using a table.

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1. (10 pts) Consider the curve

$$\mathbf{r}(t) = \left(\frac{t^3}{3} - t, t^2\right)$$

Find the equations of its tangent and normal lines at the point $(0, 3)$.

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2. (10 pts) Let $f(x, y, z) = xy^{-1} + yz^{-1} + zx^{-1}$. Find the maximum rate of change of f at the point $(1, 2, -1)$, and determine the direction in which it occurs. Express your answer as a unit vector.

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3. (10 pts) Compute

$$\oint_C \frac{x dy - y dx}{x^2 + y^2}$$

where C is the unit circle centred at $(1, 1)$.

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4. (10 pts) Say that f is a continuously differentiable function , and $\mathbf{F}$ is a continuously differentiable vector field on $\mathbf{R}^3$. Show that

$$div(f\mathbf{F}) = f\ div(\mathbf{F}) + \mathbf{F} \cdot grad(f)$$

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5. (10 pts) Find the centre of mass of a lamina whose boundary is given by $r = \cos(\theta)$(in polar coordinates) and whose density is directly proportional to the distance from the origin.

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6. (10 pts) Compute

$$\int\int_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$, and S is the part of the cone $z = \sqrt{x^2 + y^2}$ inside the cylinder $x^2 + y^2 = 4$, oriented with the inwards pointing normal.

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7. (10 pts) Prove that Green's theorem is a special case of Stokes' theorem, by identifying (x, y) in $\mathbf{R}^2$ with $(x, y, 0)$ in $\mathbf{R}^3$.

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8. (10 pts) Compute the line integral

$$\oint \mathbf{F} \cdot d\mathbf{r}$$

where $\mathbf{F} = (x, -yz, 1)$ and C is the intersection of the paraboloid $z = x^2 + y^2$ and the plane $z = 2y$, oriented counterclockwise when viewed from above.

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9. (10 pts) Compute

$$\int\!\!\int_S \mathbf{F} \cdot d\mathbf{S}$$

where $\mathbf{F} = x^2\mathbf{i} + xy\mathbf{j} + xz\mathbf{k}$, and S is the closed surface consisting of the upper hemisphere of the unit sphere $x^2 + y^2 + z^2 = 1,\ z \geq 0$ and the unit disk in the xy plane $x^2 + y^2 \leq 1,\ z = 0$.

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10. (10 pts) Compute the integral

$$\int\int_D x^2 - y^2 dA$$

where D is the region in the plane bounded by the curves $xy = 1$, $y = x - 1$, and $y = x + 1$. (Hint: Use the change of variables $x = u + v$, $y = -u + v$.)

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