HOMEWORK #1: FOURIER SERIES

Due: one minute after 11:59pm on February 1

Instructions:

- The assignment consists of *four* questions worth, respectively, 3, 2, 3, and 2 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2ZZ3a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), sections 11.1-11.2.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 12.1-12.4.

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1. Find the Fourier series of the function f defined as follows

$$f(x) = \begin{cases} 0, & -2 < x < -1, \\ x+1, & -1 \le x < 1, \\ 2x - x^2, & 1 \le x < 2. \end{cases}$$

   Plot the function f and the partial sums $S_1(x)$, $S_2(x)$, $S_5(x)$ and $S_{15}(x)$ for $x \in [-2,2]$ on the same graph using different line colours. The graph should appear as `Figure 1`. Also compute the values $S_{15}(-1)$ and $S_{15}(1)$ and save the results in the variables `Answer1` and `Answer2`, respectively. Draw a conclusion about convergence at the points $x = -1$ and $x = 1$ and save it in the variable `Answer3` (as text).

2. Compute the trigonometric approximation $f_m(x)$ of the function

$$f(x) = 1 - |x|, \quad -1 < x < 1,$$

   which is extended periodically with the period $L = 2$. Plot the functions $f_m(x)$ and $f(x)$ on $[-1,1]$ with the step $h = 0.01$ for $m = 5$ on the same graph using different line colours. The graph should appear as `Figure 2`.

3. Write the MATLAB function `[a] = LSsine(x, y, m)`, where **x** and **y** are column vectors with n elements, such that $y_i = f(x_i),\ i = 1,\ldots,n$, m is the order of interpolation, whereas **a** is a column vector with m elements representing the following sine series

$$F_{sin}(x) = a_1 \sin(\pi x) + a_2 \sin(2\pi x) + \cdots + a_m \sin(m\pi x)$$

obtained using trigonometric interpolation. Apply this function to the data obtained by evaluating the function $f(x) = x(1-x)e^{-x}$ on the interval $[0,1]$ with the step $h = 0.2$, taking $m = 5$ and $n = m+2$. Plot functions $F_{sin}(x)$ and $f(x)$ with the step $h = 0.01$ on the same graph using different line colours. The graph should appear as `Figure 3`.

4. Find the *complex* Fourier series of the function f defined as

$$f(x) = \begin{cases} -1, & -\pi < x < 0, \\ 1, & 0 \le x < \pi. \end{cases}$$

Plot the frequency spectrum $F(n) = |c_n|$, $-20 \le n \le 20$, where n is integer, of the periodic extension of the function $f(x)$. The graph should appear as `Figure 4`.