HOMEWORK #2: VECTOR CALCULUS: SCALAR AND VECTOR FIELDS

Due: one minute after 11:59pm on February 16

Instructions:

- The assignment consists of *four* questions worth, respectively, 2, 3, 3, and 2 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2ZZ3a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), sections 7.1-7.3, 7.5.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 9.1-9.7.

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1. Consider the function
$$f(x,y) = \frac{10\sin(xy)}{x^2+y^2}.$$
Use the MATLAB command `surfc` to graph the surface $f(x,y)$ for $-3 \le x \le 3$ and $-3 \le y \le 3$ with the step sizes $\Delta x = \Delta y = 0.2$. Also, using the MATLAB functions `contour` and `clabel`, plot 5 level curves of $f(x,y)$ corresponding to the values -3, -2, 0, 1, 3. Both graphs should appear as `Figure 1` (as separate subplots) with the corresponding titles.
Hint: Use the MATLAB command `subplot` to create subplots within one figure.

2. Find all first–order and second–order partial derivatives of the function
$$f(x,y) = \sqrt{1 - \frac{x^2}{4} - \frac{y^2}{9}}.$$
Plot the function $f(x,y)$ and all its partial derivatives of the first and second order for $-1 \le x \le 1$ and $-1 \le y \le 1$ with the step sizes $\Delta x = \Delta y = 0.1$ on the same figure. Use the function `subplot` to arrange the plots in 2 rows and 3 columns and put the corresponding titles for each plot. Note that, since $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$, only one of these derivatives needs to be plotted. The graph should appear as `Figure 2`.

3. Consider the function
$$f(x,y,z) = x^2 + 3y^2 + 6z.$$
Use the MATLAB command `isosurface` to plot the level surfaces of the function $f(x,y,z)$ corresponding to the levels $c = 1,4,9$ for $x,y,z \in [-2,2]$ with the step sizes $\Delta x = \Delta y = \Delta z = 0.1$. Each of the isosurfaces should have a different color and the graph should appear as `Figure 3`. Use the MATLAB command `slice` to show how the function $f(x,y,z)$ varies over the plane $x = -1$ and on the surface $z = x^2 - y^2$. Both these graphs should appear as separate subplots in `Figure 4` and should have the corresponding titles.

4. Plot the *divergence* and the *curl* of the following vector field
$$\mathbf{F}(x,y,z) = x^2 \sin(yz)\mathbf{i} + z\cos(xz^3)\mathbf{j} + ye^{5xy}\mathbf{k}$$
for $x,y,z \in [-0.1,0.1]$ with the step sizes $\Delta x = \Delta y = \Delta z = 0.01$. The corresponding graphs should appear as `Figure 5` (slices through the planes $x = 0.1$, $y = 0.1$ and $z = -0.1$) and `Figure 6` (with 3 separate subplots corresponding to the three different components of the curl vector, also as slices through the planes $x = 0.1$, $y = 0.1$ and $z = -0.1$).