HOMEWORK #4: VECTOR CALCULUS: DOUBLE AND TRIPLE INTEGRALS, INTEGRAL THEOREMS

Due: one minute after 11:59pm on March 15

Instructions:

- The assignment consists of *four* questions worth, respectively, 3, 2, 3, and 2 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2ZZ3a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), section 7.8.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 9.10-9.16.

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1. Consider a solid bounded by the graphs of the functions

$$y = x^2, \;\; y = x, \;\; z = y + 2, \;\; z = 0.$$

Using the MATLAB command `surf`, plot this solid in 3–D taking $x, y \in [-1, 1]$ and $z \in [-1, 3]$ with the step sizes $\Delta x = \Delta y = \Delta z = 0.1$. The graph should appear as `Figure 1`. Using the MATLAB command `triplequad`, find the center of mass (x_c, y_c, z_c) of this solid assuming that the density at each point is directly proportional to the distance from the *xy*–plane. Save the result in the form $[x_c \; y_c \; z_c]$ in the variable `Answer1`.
Important: In order to avoid excessive computational time, in all calls to the MATLAB command `triplequad` use the parameter `tolerance` with the value 10^{-3}.

2. Verify *Green's theorem*

$$\int\int_D \left(\frac{\partial F_2}{\partial x_1} - \frac{\partial F_1}{\partial x_2} \right) dx_1 \; dx_2 = \int_{\partial D} \mathbf{F} \cdot d\mathbf{s} \qquad (1)$$

calculating both sides of equation (1) for the vector field

$$\mathbf{F}(x,y,z) = x^3 e^{-y^2} \mathbf{i} + 2y \cos(x^3) \mathbf{j},$$

where ∂D is the ellipse parameterized as $x = 2\cos(t),\ y = 3\sin(t)$, for $0 \le t \le 2\pi$. Save the results for the left–hand and right–hand sides of (1) in the variables `Answer2` and `Answer3`, respectively.

3. Calculate both sides of the expression representing *Stokes' theorem*

$$\int\int_D \nabla \times \mathbf{F} \cdot d\mathbf{S} = \int_{\partial D} \mathbf{F} \cdot d\mathbf{s} \tag{2}$$

   for the irrotational vector field

$$\mathbf{F}(x,y,z) = \left[\frac{y}{x^2+y^2},\quad -\frac{x}{x^2+y^2},\quad 0\right]^T,\ \ (x,y) \neq (0,0)$$

   and the surface D given by the Möbius strip

$$\mathbf{M}(u,v) = \begin{bmatrix} (2+v\cos(u/2))\cos(u) \\ (2+v\cos(u/2))\sin(u) \\ v\sin(u/2) \end{bmatrix}$$

   with $0 \le u \le 2\pi$ and $-1 \le v \le 1$. Save the results for the left–hand and right–hand sides of (2) in the variables `Answer4` and `Answer5`, respectively. Draw your conclusion about the application of the Stokes' theorem to Möbius strip and save it in the variable `Answer6` (as text).
   Hint: When computing the line integral on the right–hand side of (2) use the parametrization $v = -1$ and $u = t$, where $0 \le t \le 4\pi$.

4. Use the MATLAB commads `dblquad` and `triplequad` to verify *Gauss' theorem*

$$\int\int\int_V \nabla \cdot \mathbf{F}\, dV = \int\int_S \mathbf{F} \cdot d\mathbf{S} \tag{3}$$

   for the vector field

$$\mathbf{F}(x,y,z) = \left[2x^2,\quad y^3z^2,\quad xz^3\right]^T$$

   and a sphere S of radius one centered at the origin. Save the results for the left–hand and right–hand sides of (3) in the variables `Answer7` and `Answer8`, respectively.
   Important: In order to avoid excessive computational time, in all calls to the MATLAB command `triplequad` use the parameter `tolerance` with the value 10^{-3}.