HOMEWORK #5: PARTIAL DIFFERENTIAL EQUATIONS

Due: one minute after 11:59pm on April 1

Instructions:

- The assignment consists of *three* questions worth, respectively, 3, 3, and 4 points.
- Submit your assignment *electronically* to the Email address specific to your last name as indicated on the course website; the file containing your assignment must be named `Name_0XXXXXX_hwN.m`, where "`Name`" is your last name, "`XXXXXX`" is your student ID number, and "`N`" is the consecutive number of the assignment; hardcopy submissions will not be accepted.
- It is obligatory to use the *current* MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH2ZZ3a/template.m`; submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.
- All graphs should contain suitable titles and legends.
- Reference:
  1. "**Numerical Mathematics**" by M. Grasselli and D. Pelinovsky (Jones and Bartlett, 2008), sections 10.1, 10.3-10.5.
  2. "**Advanced Engineering Mathematics**" by D.G. Zill and M.R. Cullen (Jones and Bartlett, 3rd edition), sections 13.1-13.5.

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1. Consider the nonlinear *heat equation*

$$u_t = u_{xx} - u(1-u^2), \quad -1 < x < 1, \ t > 0,$$

subject to the boundary conditions $u(-1,t) = u(1,t) = 0$ and the initial condition $u(x,0) = 1 - x^2$. Approximate the solution of this problem with an *explicit method*. Discretize the domain with the step size $h = 0.1$ and time step $\tau = 0.5h^2$. Plot the solution $u(x,t)$ versus x for times $t = 0,\ 0.25,\ 0.5,\ 0.75,\ 1$. The graph should appear as `Figure 1`.

2. Use an *explicit method* defined as

$$\frac{u_{k,l+1} - 2u_{k,l} + u_{k,l-1}}{\tau^2} = \frac{u_{k+1,l} - 2u_{k,l} + u_{k-1,l}}{h^2} + O(\tau^2, h^2),$$

where the indices k and l correspond to the discretization in x and t, respectively, in order to approximate the solution of the *wave equation*

$$u_{tt} = u_{xx}, \quad 0 < x < 1, \ t > 0,$$

subject to the boundary conditions $u(0,t) = u(1,t) = 0$ and the initial conditions $u(x,0) = \sin(2\pi x)$ and $u_t(x,0) = 0$. Use an equispaced grid with the step size $h = 0.025$ and the time step $\tau = 0.025$. Plot the solution $u(x,t)$ versus x for times $t = 0,\ 0.125,\ 0.25,\ 0.375,\ 0.5$. The graph should appear as `Figure 2`.

3. Consider the boundary–value problem for the *Poisson equation*

$$u_{xx} + u_{yy} = f(x,y),$$

where

$$f(x,y) = x(1-x)y(1-y), \quad 0 < x < 1, \; 0 < y < 1,$$

subject to the zero Dirichlet boundary conditions. Use an equispaced grid with the step sizes $h_x = h_y = 0.25$ and compute the solution of the problem using the *finite–difference method* defined as

$$\frac{u_{k+1,m} - 2u_{k,m} + u_{k-1,m}}{h_x^2} + \frac{u_{k,m+1} - 2u_{k,m} + u_{k,m-1}}{h_y^2} = f(x_k, y_m),$$

where the indices k and m correspond to the discretization in x and y, respectively. Plot the solution $u(x,y)$ using the MATLAB command `surf`. The graph should appear as `Figure 3`.