TEST #1 (VERSION 4)

19:00 — 20:15, February 4, 2010

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- This text paper consists of 8 pages (including this one). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator.
- There are 16 multiple–choice questions worth 1 mark each (no part marks).
- The questions must be answered on the COMPUTER CARD with an HB PENCIL.
- Make sure to indicate the **test version** and your **student number**.
- Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing).
- Only the McMaster Standard Calculator Casio FX991MS is allowed

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Computer Card Instructions

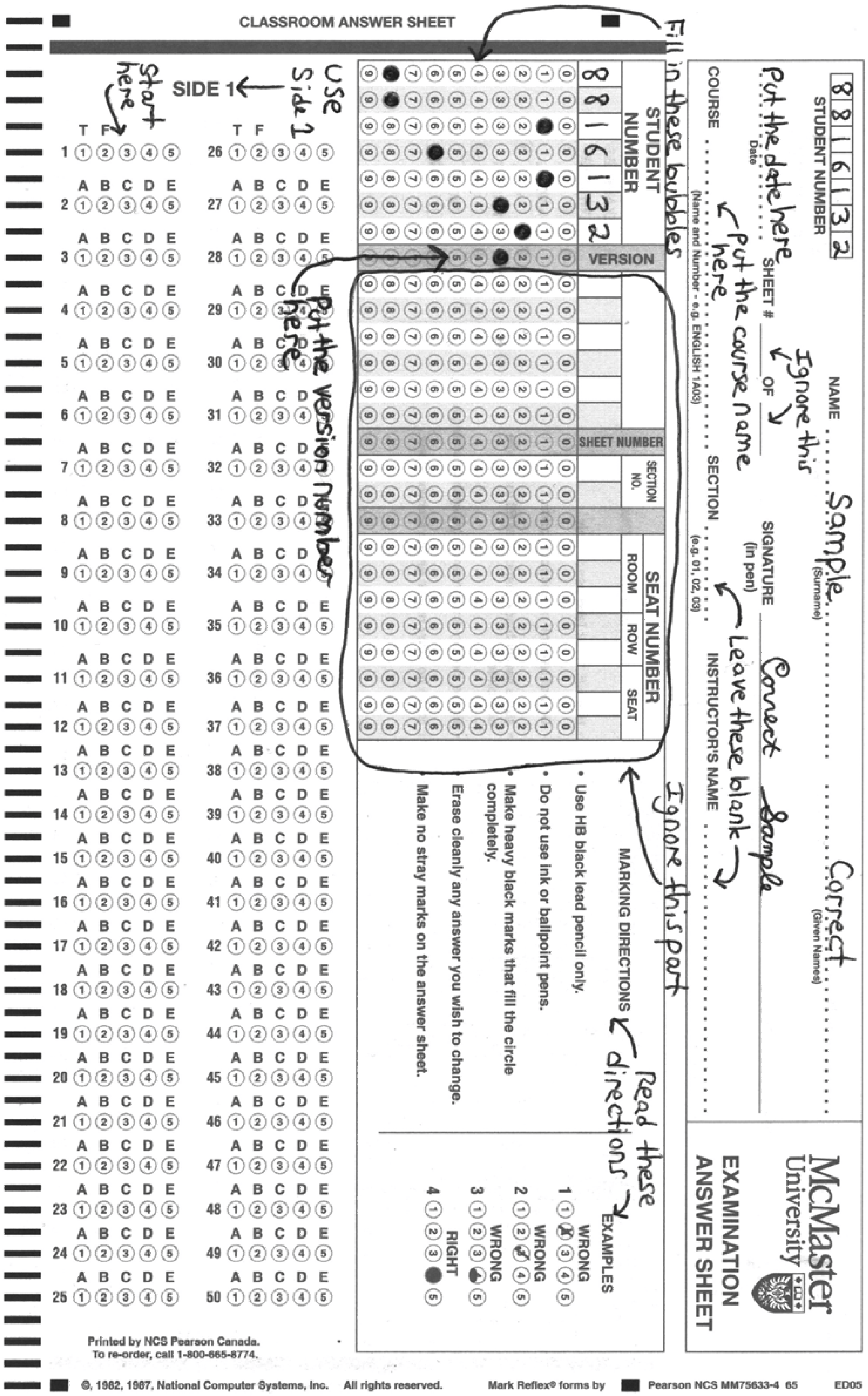
NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will NOT be sensed. Erasures must be thorough or the scanner may still sense a mark. Do NOT use correction fluid on the sheets. Do NOT put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet MUST be signed in the space marked SIGNATURE.
2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only ONE choice from the alternatives (A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked “1”.

See Figure on the next page for additional information on filling computer cards.

Sample of how to fill out the computer card:



1. A particle has the position $\mathbf{r}(t)$ at time t given by $\mathbf{r}(t) = 3\,\mathbf{i} - 2t\,\mathbf{j} + t^2\,\mathbf{k}$. At time $t = 0$ its acceleration has the normal component

   (a) 0

   (b) -2

   (c) 2

   (d) -2 **j**+2t **k**

   (e) $\frac{1}{2}$

   HINT: the normal component of acceleration can be computed with the formula $a_N = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|}$

2. The level curves of the function $f(x,y) = \ln(x+y^2)$ are

   (a) $\ln(x) = C - \ln(y^2)$

   (b) $y = (e^C + x)^{1/2}$

   (c) $y = (e^C - x)^{1/2}$

   (d) $x = e^{C-y^2}$

   (e) $x = e^C - y^2$

3. If $w(u,v)$ is a differentiable function and $\frac{\partial w}{\partial u}\Big|_{(u,v)=(-1,2)} = 1$, $\frac{\partial w}{\partial v}\Big|_{(u,v)=(-1,2)} = 2$, $u(x,y) = xy$, and $v(x,y) = x^2 + y^2$, then $\frac{\partial w}{\partial y}\Big|_{(x,y)=(1,-1)}$ is equal to

   (a) 3

   (b) -3

   (c) -1

   (d) 0

   (e) 5

4. Suppose $z(x,y) = e^{x/y}$, then $\frac{\partial^2 z}{\partial x \partial y}$ is equal to

   (a) $e^{x/y}$

   (b) $-\frac{2}{y^3}e^{x/y}$

   (c) $-\frac{1}{y^2}e^{x/y} - \frac{x}{y^3}e^{x/y}$

   (d) $\left(\frac{x}{y}\right)^2 e^{x/y}$

   (e) $\frac{1}{y}e^{x/y} - \frac{x}{y^2}e^{x/y}$

5. Suppose $\mathbf{r}(t) = 2\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}$ is the position vector of a moving particle. What is the speed of the particle at time $t = 2$?

   (a) 5
   (b) $\sqrt{1+4t^2}$
   (c) $\sqrt{17}$
   (d) 2
   (e) none of the above

6. Given the position vector $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$ of a moving particle, find the acceleration of the particle at time t=1

   (a) $\mathbf{a}(t) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$
   (b) $\mathbf{a}(t) = 2\,\mathbf{j} + 6\,\mathbf{k}$
   (c) $\mathbf{a}(t) = \mathbf{i} + \mathbf{j} + \mathbf{k}$
   (d) $\mathbf{a}(t) = 2\,\mathbf{j} + 12\,\mathbf{k}$
   (e) none of the above

7. A shell is fired from the ground level with an initial speed of 200 m/s at an angle of elevation of 30^o. The vector function that describes the trajectory of the shell is:

   (a) $\mathbf{r}(t) = 100\sqrt{3}t\,\mathbf{i} + (100t - 5t^2)\,\mathbf{j}$
   (b) $\mathbf{r}(t) = 100\sqrt{3}t\,\mathbf{i} + (100t - 10t^2)\,\mathbf{j}$
   (c) $\mathbf{r}(t) = 100\sqrt{3}t\,\mathbf{i} + 100t\,\mathbf{j}$
   (d) $\mathbf{r}(t) = 100\sqrt{3}t\,\mathbf{i} - 5t^2\,\mathbf{j}$
   (e) none of the above

   HINT: use $\mathbf{g} = -10\,\mathbf{j}$ and assume that the origin coincides with the point where the shell is fired from

8. The position vector of a moving particle at time t is $\mathbf{r}(t) = 3\cos(2t)\,\mathbf{i} + 4\,\mathbf{j} + 3\sin(2t)\,\mathbf{k}$. The curvature of the path is

   (a) $\frac{1}{6}$
   (b) $\frac{1}{3}$
   (c) $\frac{1}{5}$
   (d) 3
   (e) $22^{1/2}$

   HINT: the curvature can be computed with the formula $\kappa = \frac{\|\mathbf{r}'(t)\times\mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

9. The function $f(x) = |x|$, $-\pi < x < \pi$, can be represented as the following series

   (a) $f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\cos(nx)$

   (b) $f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\cos(nx)$

   (c) $f(x) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n-1}{n^2}\sin(nx)$

   (d) $f(x) = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{(n+1)}}{n}\cos(nx)$

   (e) none of the above

10. Given the function $f(x) = e^{-x}$, its complex Fourier series on the interval $[-1,1]$ is

   (a) $f(x) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1+in\pi}e^{in\pi x}$

   (b) $f(x) = \frac{e-e^{-1}}{2}\sum_{n=-\infty}^{\infty}\frac{1}{1+in\pi}e^{in\pi x}$

   (c) $f(x) = \frac{e-e^{-1}}{2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1+n\pi}e^{in\pi x}$

   (d) $f(x) = \frac{e-e^{-1}}{2}\sum_{n=-\infty}^{\infty}\frac{(-1)^n}{1+in\pi}e^{in\pi x}$

   (e) $f(x) = \frac{1}{2}\sum_{n=-\infty}^{\infty} e^{in\pi x}$

11. The vector function **r**(t) that satisfies the conditions $\mathbf{r}''(t) = 12t\,\mathbf{i} - 3t^2\,\mathbf{j} + 2\,\mathbf{k}$, $\mathbf{r}'(0) = \mathbf{j}$, $\mathbf{r}(0) = 2\,\mathbf{j} - \mathbf{k}$ is

   (a) $\mathbf{r}(t) = 2t^3\,\mathbf{i} + (2 + t - \frac{t^4}{4})\,\mathbf{j} + (t^2 - 1)\,\mathbf{k}$

   (b) $\mathbf{r}(t) = 6t^2\,\mathbf{i} + (1 - t^3)\,\mathbf{j} + (2t)\,\mathbf{k}$

   (c) $\mathbf{r}(t) = 2t^3\,\mathbf{i} + (t - \frac{t^3}{4})\,\mathbf{j} + (t^2)\,\mathbf{k}$

   (d) $\mathbf{r}(t) = 6t^2\,\mathbf{i} + (t^3)\,\mathbf{j} + (2t)\,\mathbf{k}$

   (e) none of the above

12. The length of the curve traced by the vector $\mathbf{r}(t) = 3t\,\mathbf{i} + \sqrt{7}t\,\mathbf{j} + \frac{2}{3}\,\mathbf{k}$ on the interval $1 \le t \le 2$ is

   (a) 16

   (b) $3 + \sqrt{7}$

   (c) $(3 + \sqrt{7})^2$

   (d) 4

   (e) none of the above

13. Assuming that the vector `x` contains the values of the function f at N equispaced points, the vector of the discrete Fourier coefficients of the function f can be computed in MATLAB as follows

    (a) `c = Fourier( x )`

    (b) `c = DFT( x )`

    (c) `c = fft( x )`

    (d) `c = FourierTransform( x )`

    (e) `c = sin(x) + 1i * cos(x)`

14. Which of the following sets of functions is *not* an orthogonal set on the interval indicated?

    (a) $\{\sin(x), \sin(2x), \sin(3x), \dots\}$ on $[-\pi, \pi]$

    (b) $\{\cos(x), \cos(2x), \cos(3x), \dots\}$ on $[0, 2\pi]$

    (c) $\{e^{ix}, e^{2ix}, e^{3ix}, \dots\}$ on $[0, \pi]$

    (d) $\{\cos(2x), \cos(4x), \cos(6x), \dots\}$ on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

    (e) $\{\sin(2x), \sin(4x), \sin(6x), \dots\}$ on $[0, \pi]$

15. You are given the following function

$$f(x) = \begin{cases} -x - \dfrac{3}{4} & \text{for } -1 \le x < 0, \\ -\left(x - \dfrac{1}{2}\right)^2 & \text{for } 0 \le x < 1. \end{cases}$$

    At the point of discontinuity $x = 0$ its Fourier series expansion assumes the value

    (a) -0.5

    (b) 0.5

    (c) 1

    (d) 0

    (e) -0.25

16. Consider the function $f(x) = \sin(2x)\cos(2x)$ on the interval $[-\pi, \pi]$. The Fourier series of this function $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nx) + b_n \sin(nx)]$ has the following *nonzero* coefficients

    (a) $a_n = b_n = \frac{1}{n^2}$, $n > 0$

    (b) $a_n = \frac{1}{n^2}$, $n > 0$

    (c) $a_2 = b_2 = 1$

    (d) $a_4 = 1$

    (e) $b_4 = \frac{1}{2}$

(rough work)

(rough work)