TEST #2 (VERSION 4)

19:00 — 20:15, March 16, 2010

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- This text paper consists of 8 pages (including this one). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator.
- There are 18 multiple–choice questions worth 1 mark each (no part marks).
- The questions must be answered on the COMPUTER CARD with an HB PENCIL.
- Make sure to indicate the **test version** and your **student number**.
- Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing).
- Only the McMaster Standard Calculator Casio FX991MS is allowed

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Computer Card Instructions

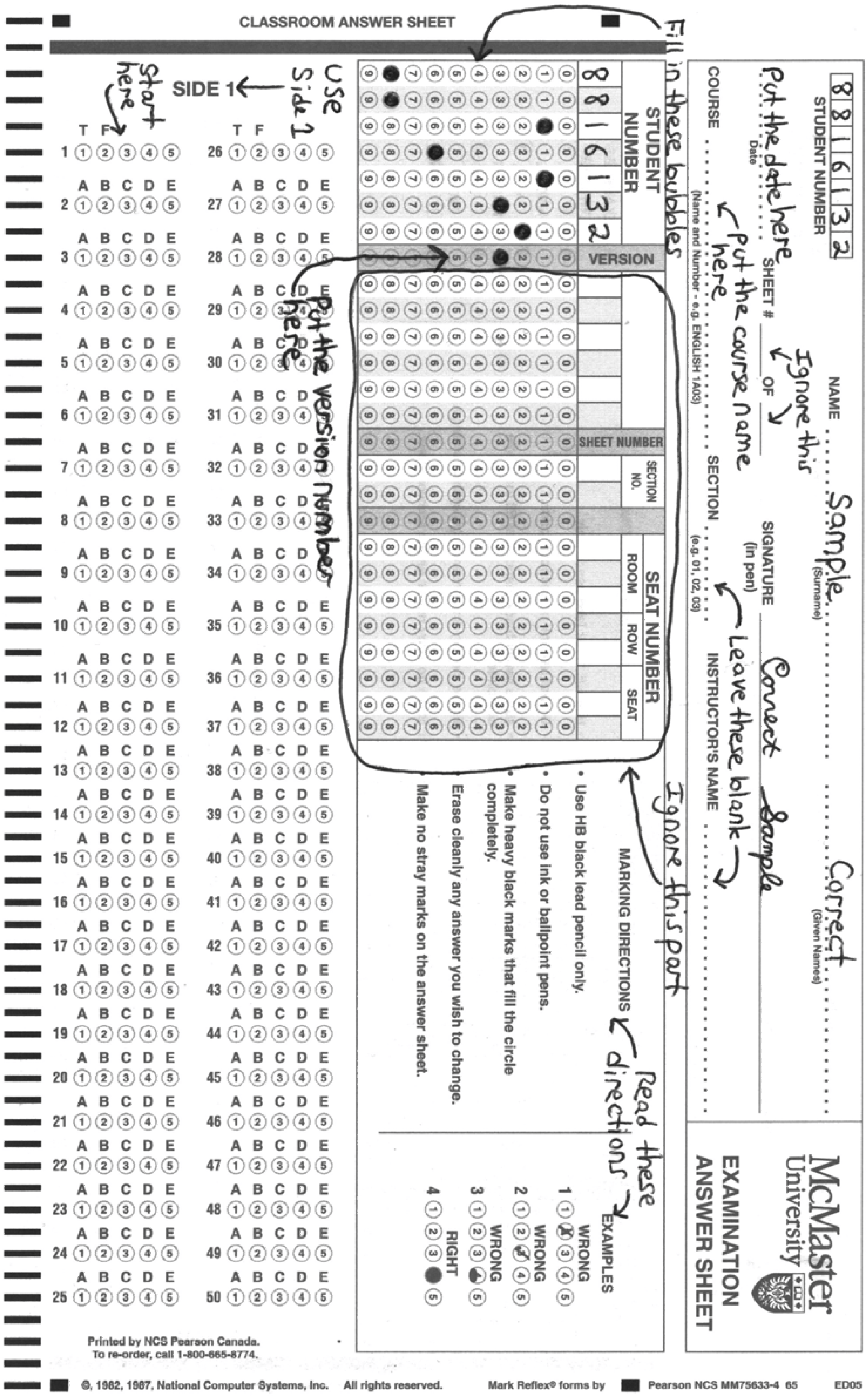
NOTE: IT IS YOUR RESPONSIBILITY TO ENSURE THAT THE ANSWER SHEET IS PROPERLY COMPLETED: YOUR EXAMINATION RESULT DEPENDS UPON PROPER ATTENTION TO THESE INSTRUCTIONS

The scanner, which reads the sheets, senses the shaded areas by their non-reflection of light. A heavy mark must be made, completely filling the circular bubble, with an HB pencil. Marks made with a pen or felt-tip marker will NOT be sensed. Erasures must be thorough or the scanner may still sense a mark. Do NOT use correction fluid on the sheets. Do NOT put any unnecessary marks or writing on the sheet.

1. Print your name, student number, course name, and the date in the space provided at the top of Side 1 (red side) of the form. Then the sheet MUST be signed in the space marked SIGNATURE.
2. Mark your student number in the space provided on the sheet on Side 1 and fill in the corresponding bubbles underneath.
3. Mark only ONE choice from the alternatives (A,B,C,D,E) provided for each question. If there is a True/False question, enter response of 1 (or A) as True, and 2 (or B) as False. The question number is to the left of the bubbles. Make sure that the number of the question on the scan sheet is the same as the question number on the test paper.
4. Pay particular attention to the Marking Directions on the form.
5. Begin answering questions using the first set of bubbles, marked “1”.

See Figure on the next page for additional information on filling computer cards.

Sample of how to fill out the computer card:



1. The surface S has the vector description $\mathbf{r}(s,t) = st\,\mathbf{i} + (s^2 - t^2)\,\mathbf{j} + (s^2 + t^2)\,\mathbf{k}$. The normal line through the point $\mathbf{r}(1,1)$ has parametric description (with parameter p)

   (a) $(1+8p)\,\mathbf{i} + (2-4p)\,\mathbf{k}$

   (b) $\mathbf{i} + (2+4p)\,\mathbf{j}$

   (c) $\frac{p}{\sqrt{2}}\,\mathbf{i} - \frac{p}{\sqrt{2}}\,\mathbf{k}$

   (d) $(1+4p)\,\mathbf{i} + 4p\,\mathbf{j}$

   (e) $(1+4p)\,\mathbf{i} + (-2+4p)\,\mathbf{k}$

2. Consider the vector function $\mathbf{F}(x,y,z) = \mathbf{i} + y\,\mathbf{k}$ and the surface $S : z = xy$ above the triangle $x \geq 0$, $y \geq 0$, $x + y \leq 4$ oriented so that the normal makes an acute angle with the vector $\mathbf{k}$. The flux of $\mathbf{F}$ through S, i.e., $\iint_S \mathbf{F} \cdot d\mathbf{S}$, is equal to

   (a) $\frac{1}{2}$

   (b) 0

   (c) $-\frac{1}{2}$

   (d) $\frac{8}{3}$

   (e) $-\frac{8}{3}$

3. The surface S is the part of the paraboloid $z = 4 - x^2 - y^2$ above the (x,y) plane with the normal vector pointing away from that plane. Consider the vector function $\mathbf{F}(x,y,z) = 2y\,\mathbf{i} + x\,\mathbf{j} + z\,\mathbf{k}$. The integral $\iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$ is equal to

   (a) 0

   (b) 4π

   (c) -4π

   (d) $4\pi^2$

   (e) -1

4. The centroid of the constant–density solid described by the relation $x^2 + y^2 \leq z \leq 1$ is located at

   (a) $\left(0,0,\frac{\pi}{3}\right)$

   (b) $\left(0,0,\frac{1}{3}\right)$

   (c) $\left(0,0,2\frac{\pi}{3}\right)$

   (d) $\left(0,0,\frac{z^2}{2}\right)$

   (e) $\left(0,0,\frac{2}{3}\right)$

5. The integral $\int_0^1 \int_{\sqrt{x}}^1 \cos(y^3)\,dy\,dx$ equals to

   (a) $\frac{\cos 1}{3}$
   (b) $\frac{\sin 1}{3}$
   (c) $\frac{\sin 1}{3} - \frac{2}{3}$
   (d) $\frac{\cos 1}{3} - \frac{2}{3}$
   (e) none of the above

   HINT: Evaluate the integral by changing the order of integration.

6. The integral $\iint_D \sqrt{x^2+y^2}\,dA$, where D is the region bounded by the circle $x^2+(y-1)^2=1$, is

   (a) 0
   (b) $\frac{64}{9}$
   (c) $\frac{32}{9}$
   (d) $\frac{\pi}{3}$
   (e) none of the above

   HINT: Evaluate the integral using polar coordinates.

7. A uniform plate with mass density $\rho(x,y) = 2\ g/cm^3$ occupies the region bounded by the semi-circles $y=\sqrt{4-x^2}$, $y=\sqrt{1-x^2}$ and such that $y \geq 0$. Then the centre of mass $(\overline{x},\overline{y})$ of the plate is at

   (a) $(\frac{28}{3\pi}, 0)$
   (b) $(0, \frac{28}{\pi})$
   (c) $(\frac{28}{9\pi}, 0)$
   (d) $(0, \frac{28}{9\pi})$
   (e) none of the above

8. The surface area of the spherical cap described by $x^2+y^2+z^2=4$ and $1 \leq z \leq 2$ is

   (a) 2π
   (b) -2π
   (c) 4π
   (d) $4\pi(2-\sqrt{3})$
   (e) $4\pi(\sqrt{3}-2)$

9. The value of the path integral $\oint_C \ln(1+y)\,dx - \frac{xy}{1+y}\,dy$, where C is the triangle with the vertices $(0,0)$, $(1,1)$, $(0,1)$ (in the order listed), is

   (a) 1
   (b) $\frac{1}{2}$
   (c) $-\frac{1}{2}+\ln 2$
   (d) $\frac{1}{2}-\ln 2$
   (e) $-\frac{1}{2}$

   HINT: Use Green's Theorem.

10. Consider the vector field $\mathbf{F} = xz\,\mathbf{i} + yz\,\mathbf{j} + xy\,\mathbf{k}$. The curl $\nabla \times \mathbf{F}$ of this vector field is

(a) $(x-y)\,\mathbf{i} + (x-y)\,\mathbf{j} + (x-y)\,\mathbf{k}$

(b) $(x-y)\,\mathbf{i} + (x-y)\,\mathbf{j}$

(c) $(x+y)\,\mathbf{i} + (x+y)\,\mathbf{j}$

(d) $xz\,\mathbf{i} + yz\,\mathbf{j} + xy\,\mathbf{k}$

(e) $z\,\mathbf{j} + y\,\mathbf{k}$

11. Assuming that $\mathbf{F}$ is a *vector* field and f is a *scalar* field, which of the following sets of statements is true for all differentiable $\mathbf{F}$ and f?

(a) $\nabla \cdot f = 0$ and $\nabla \cdot \mathbf{F} = 0$

(b) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ and $\nabla \cdot \nabla f = 0$

(c) $\nabla \cdot (\nabla \times \mathbf{F}) = 0$ and $\nabla \times \nabla f = \mathbf{0}$

(d) $\nabla \times (\nabla \times \mathbf{F}) = \mathbf{0}$ and $\nabla(\nabla \cdot \mathbf{F}) = \mathbf{0}$

(e) none of the above

12. The integral $\oint_C (3x^2 + y^2)\,dx - 3xy\,dy$, where the curve C (oriented counterclockwise) is the boundary of the region bounded by the graphs of $y = x^2$ and $y = 1$, equals to

(a) 4

(b) -4

(c) 0

(d) -1

(e) none of the above

13. Which of the following is *false* in regard to the vector field $\mathbf{F}(x,y) = (y^2+5)\,\mathbf{i} + (2xy-8)\,\mathbf{j}$?

(a) It is a conservative vector field

(b) $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$, where C is the unit circle

(c) $f(x,y) = xy^2 - 8y + 5x + 5$ is a potential for $\mathbf{F}$

(d) $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ for any path C that connects the points $(\frac{1}{6}, 1)$ and (1,2)

(e) none of the above

14. The double integral $\iint\limits_D 2xy\,dA$, where D is the region bounded by the graphs of $x = 0$, $y = -1$ and $x + y = 5$, is equal to

(a) 36

(b) 56

(c) 81

(d) 72

(e) none of the above

15. MATLAB command which evaluates a double integral over a rectangle is

    (a) `double_int(...)`
    (b) `dblquad(...)`
    (c) `Double_Int(...)`
    (d) `int2(...)`
    (e) MATLAB does not have such a command

16. A surface plot of the function $f(x,y) = x^2 + y^2$, where $x, y \in [-1, 1]$, can be obtained with the following set of MATLAB commands

    (a)
    ```
    x = -1:0.01:1; y = x;
    z = x^2+y^2;
    surf(x,y,z);
    ```
    (b)
    ```
    x = -1:0.01:1; y = x;
    z = x.^2+y.^2;
    surf(x,y,z);
    ```
    (c)
    ```
    x = -1:0.01:1; y = x;
    [X,Y] = meshgrid(x,y);
    z = X^2+Y^2;
    plot3(x,y,z);
    ```
    (d)
    ```
    x = -1:0.01:1; y = x;
    [X,Y] = meshgrid(x,y);
    z = X^2+Y^2;
    surf(x,y,z);
    ```
    (e)
    ```
    x = -1:0.01:1; y = x;
    [X,Y] = meshgrid(x,y);
    z = X.^2+Y.^2;
    surf(x,y,z);
    ```

17. The temperature of a flat plate is described by the expression $T(x,y) = 4x^2 + 2y^4$. Which of the following functions describes the path of a heat–seeking insect starting at the point $(1,1)$?

    (a) $y = \frac{1}{1-\ln x}$
    (b) $y = x$
    (c) $y^2 = \frac{1}{1-\ln x^2}$
    (d) $y = x^2$
    (e) $x = 4y^3$

18. Consider the function $z = x^{-1} + 2y^2 + \frac{1}{2}$. The plane tangent to the graph of this function at the point $(2,1,3)$ is given by the expression

    (a) $-x + 16y - 4z = 2$
    (b) $-x + y - z = 2$
    (c) $-x + 16y - z = 7$
    (d) $-3x + 7y - 4z = 2$
    (e) none of the above

(rough work)

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