HOMEWORK #2

Due: February 7 (Wednesday) by midnight

Instructions:

- The assignment consists of *two* questions, worth 3 and 5 points.
- Submit your assignment *electronically* (via Email) to the instructor; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH3Q03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. Write a MATLAB code which will perform the following calculations:
   (a) will construct the Hilbert matrix with entries $A_{ij} = \frac{1}{i+j-1}$, $i, j = 1, \ldots, N$ for any given dimension N,
   (b) calculate the norms $\|A\|_1$, $\|A\|_2$, $\|A\|_\infty$ and $\|A\|_f$ for any given square matrix A (do not use the MATLAB function `norm`, but write your own function instead; your can use the function `norm` to check if your result are correct),
   (c) calculate and write out all of the above norms for the Hilbert matrix with $N = \{25, 50, 75, 100, \ldots, 500\}$; plot these results on a single graph (using linear coordinates and different line colors for different norms);

   (3 points)

2. Consider the function $f(x) = \sin(x) - \frac{1}{2}$ and write a MATLAB code that will perform the following tasks:
   (a) draw a plot of this function in the interval $\Omega = [x_1, x_2]$, where $x_1 = 0$ and $x_2 = \frac{\pi}{2}$ (use red color for the line),
   (b) use the three codes posted on the course website (i.e., `bisect.m`, `secant.m` and `fixpoint.m`) to find the root x_0 of the equation $f(x) = 0$ in the interval Ω; use x_1 and, if needed, x_2 to initialize the iterations,
   (c) write you own MATLAB code that that will implement Newton's method to solve this problem,
   (d) draw a log–log plot of the quantity $|x_n - x_0|$ as a function of n (the iteration index) for the results obtained using the four methods mentioned above; use different symbols to mark data points corresponding o the different methods.

   Note that you may need to modify the MATLAB codes provided to ensure that they return suitable data. When using the fixed–point iterations, transform the problem to the form $g(x) = x$, where $g(x) = -\sin(x) + \frac{1}{2} + x$.
   (5 points)