HOMEWORK #3

Due: February 28 (Wednesday) by midnight

Instructions:

- The assignment consists of *two* questions, worth 4 points each.
- Submit your assignment *electronically* (via Email) to the instructor; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH3Q03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. You are given the following function

$$F(x,y) = \cos(y)^2(x-1)^4 + 100(1-\cos(y))^2. \tag{1}$$

   (a) Using the command `contourf` plot this function in the domain $D = [-5,5] \times [-5,5]$ using the step size $h = 0.1$,

   (b) use Newton's method to find the critical points (x_1^*, y_1^*) and (x_2^*, y_2^*) of the function $F(x,y)$ starting from the following two initial guesses:

$$(x_1^0, y_1^0) = (2,1),$$
$$(x_2^0, y_2^0) = (5,3);$$

   check definiteness of the Hessian of F at these two critical points in order to determine their type (local minimum, maximum, or saddle),

   (c) using two separate figures plot the history of the quantity $|F(x^k,y^k) - F(x^*,y^*)|$ and the 2–norm of the gradient $\|\nabla F(x^k,y^k)\|_2$ as a function of the iteration count k in the two cases.

   (4 points)

2. Your are given the function $f(x) = x\cos(x^2)$ in the interval $\Omega = [-\pi, \pi]$ which is discretized using a uniformly distributed grid with $N+1$ grid points (take $x_0 = -\pi$ and $x_N = \pi$).

   (a) consider the Vandermonde approach to interpolation and determine the value of N for which the condition number κ (obtained using MATLAB function `cond`) of the interpolation matrix exceeds the threshold $E = 10^6$; write out this value of N,

   (b) calculate the Lagrange interpolating polynomials in the case when $N = 4$ and plot them using different colors and the step size $h = x_{i+1} - x_i = \frac{\pi}{50}$,

   (c) using two separate figures plot the function $f(x)$ in the interval Ω together with the interpolating polynomials constructed using the Lagrange polynomials with $N = 4$ and $N = 16$; use the same h as before,

(d) using the Lagrange interpolating polynomials constructed above determine and print out the interpolation error at $x = 0.01$ and with $N = 2, 4, \ldots, 10$; repeat this using the function $g(x) = |f(x)|$; what explains the different behavior of the error in the two cases?

(4 points)