# HOMEWORK #4

Due: March 14 (Wednesday) by midnight

**Instructions:**

- The assignment consists of *two* questions, worth 4 points each.
- Submit your assignment *electronically* (via Email) to the instructor; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH3Q03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. You are given the following set of data points

| $x_i$ | -1 | -0.75 | -0.5 | -0.25 | 0.0 | 0.25 | 0.5 | 0.75 | 1 |
|---|---|---|---|---|---|---|---|---|---|
| $y_i$ | 0 | 0 | 0 | -1 | -0.5 | -1 | 0 | 0 | 0 |

Construct *cubic* splines interpolating these data point making the following assumptions as regards the two additional conditions required to close the system:

(a) $s_0 = s_n = 0$ ("natural" splines),

(b) slopes at the endpoints are given by $f'(x_0) = 3$ and $f'(x_n) = -3$,

(c) the end cubics approach parabolas at their extremities,

(d) the values of $s_0$ and $s_n$ are extrapolated linearly from the values at the neighboring nodes, i.e., $s_1, s_2$ and $s_{n-2}, s_{n-2}$, respectively.

Plot the four interpolants together with the original data on a single plot using different colors (make sure to provide a legend!).
HINT — see the four boxed points on page 172 in the textbook by Gerald & Wheatley for relevant formulas; you may start by modifying the code `spline_01.m` posted on the course webpage.
*(4 points)*

2. Your are given a vector $\mathbf{u} = [1,\ 2]^T \in \mathbb{R}^2$. Consider a family of basis vectors in $\mathbb{R}^2$ of the form

$$\begin{aligned} \mathbf{e}_1 &= [1,\ 0]^T, \\ \mathbf{e}_2 &= [\cos(\theta),\ \sin(\theta)]^T, \end{aligned} \tag{1}$$

where $\theta \in (0, \pi/2]$. Implement an algorithm that, for a given value of $\theta$, will perform decomposition of the vector $\mathbf{u}$ in the basis $\{\mathbf{e}_1, \mathbf{e}_2\}$, i.e., will find the coefficients $u_1 = u_1(\theta)$ and $u_2 = u_2(\theta)$ such that $\mathbf{u} = u_1(\theta)\mathbf{e}_1 + u_2(\theta)\mathbf{e}_2$. Then

(a) on three separate figures plot the vectors of the primal and dual bases, $\{\mathbf{e}_1, \mathbf{e}_2\}$ and $\{\mathbf{e}^1, \mathbf{e}^2\}$ respectively, for the following values of the angle $\theta = \pi/12, \pi/4, \pi/2$; use the same aspect ratio for the horizontal and vertical axes (command `axis equal`),

(b) plot the coefficients $u_1(\theta)$ and $u_2(\theta)$ on a single plot as a function of $\theta \in (0, \pi/2]$ using a step size of $\Delta\theta = 0.01$,

(c) verify that the decomposition $\mathbf{u} = u_1(\theta)\mathbf{e}_1 + u_2(\theta)\mathbf{e}_2$ is indeed correct for three values of $\theta$, e.g., $\theta = \pi/12, \pi/4, \pi/2$; what happens for $\theta = 0$?

HINT — note that for *nonorthogonal* bases, such as (1), the coefficients are calculated as $u_1 = (\mathbf{u}, \mathbf{e}^1)$ and $u_2 = (\mathbf{u}, \mathbf{e}^2)$, where $(\cdot, \cdot)$ denotes the inner product in $\mathbb{R}^2$, whereas $\mathbf{e}^1$ and $\mathbf{e}^2$ are elements of the *dual* basis defined so that $(\mathbf{e}_i, \mathbf{e}^j) = \delta_{ij}$ ($\delta_{ij}$ is the Kronecker symbol).

*(4 points)*