# HOMEWORK #6

Due: April 11 (Wednesday) by midnight

**Instructions:**

- The assignment consists of *two* questions worth 4 points each.
- Submit your assignment *electronically* (via Email) to the instructor; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH3Q03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. You are given the following integral $\int_2^4 (x^8 + x^3 - x + 2)\,dx$. Evaluate this integral numerically using Simpson's "1/3" rule and the Gaussian quadrature subdividing the interval $[a,b]$ into $2,4,\ldots,100$ subintervals. Then

   (a) determine the *relative* errors obtained with the two methods and plot them as a function of the inverse of the number of points used to evaluate the quadratures; use the log–log coordinates;

   (b) using a least–squares fit, determine empirically the order of accuracy of Simpson's "1/3" rule,

   (c) explain why the error in Gaussian integration stops to decrease at some point.

   HINT — Use the function `lgwt` posted on the course website to determine the quadrature points and weights for the Gaussian integration.
   *(4 points)*

2. Your are given the following definite integral $I = \int_0^1 x^3 e^{-x^2}\,dx = \frac{1}{2} - e^{-1}$. Calculate this integral numerically using the following values of the step size $h_k = \frac{h}{2^{k-1}}$, where $k = 1,2,\ldots,10$ and $h = 0.5$. Use the following quadratures:

   (a) Simpson's "1/3 rule",

   (b) Romberg Integration (i.e., Richardson interpolation applied to the above).

   Plot on a single figure the relative errors of integration defined as $\left|\frac{I_{approx} - I_{exact}}{I_{exact}}\right|$ and obtained in the two cases as a function of the number of grid points.
   *(4 points)*