MATH 3Q03
Numerical Interpolation and Approximation Theory
(Winter 2007)

Time & Place:

- Lectures — 13:30–14:20 on Mondays, Wednesdays & Thursdays in BSB/122
- Computer Labs — 12:30-13:20 on Mondays in ABB/166

Instructor: Dr. Bartosz Protas
Office HH 326, Ext. 24116, Email: bprotas@mcmaster.ca
Office Hours: Thursday, 10:30-12:30

Teaching Assistant: None

Course Webpage: http://www.math.mcmaster.ca/~bprotas/MATH3Q03a

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Outline of the Course: This course will discuss both theoretical and practical aspects of numerical interpolation and approximation. Such techniques form the core of Numerical Analysis and are the basis for solution of many important problems. We will review the relevant mathematical background and will show how it can be used to construct practical algorithms. Actual implementation of these algorithms will be addressed using MATLAB. Our focus will be on applications to numerical differentiation and integration of functions. We will also review certain additional, closely related, topics such as solution of nonlinear equations and unconstrained optimization.

Topics: [the actual order may be different; characters in brackets represent the reference ("I" means that the material will be provided by the instructor)]

1. Introduction & Review of the Background Material
   - basic definitions [GP],
   - introduction to MATLAB [GP],
   - properties of polynomials [GP],
   - solution of systems of equations: linear & nonlinear [GW, GP].
2. Interpolation
   - Vandermonde, Lagrange & Newton interpolation [GP],
   - error analysis: Runge phenomenon [GP, I],
   - splines [GW],
   - Chebyshev interpolation [I],
   - trigonometric interpolation [GP].
3. Approximation
   - best approximations and orthogonal projections [I],
   - systems of orthogonal polynomials [I],
   - finding best approximations [GP].
4. Numerical Differentiation and Integration
   - derivatives via finite differences, error analysis [GP],
   - Richardson extrapolation [GP],
   - numerical quadratures, error analysis [GP],

- spectral differentiation [I],
- Gaussian quadratures [GP].

5. Special Topics
   - relation between interpolation and approximation [I],
   - collocation vs. Galerkin methods for differential equations [I],
   - proper orthogonal decomposition [I].

Primary Reference:

[GP] M. R. Grasselli & D. E. Pelinovsky, "Numerical Interpolation & Approximation Theory", courseware, McMaster (2007).

Secondary References:

[GW] C. F. Gerald & P. O. Wheatley, "Applied numerical analysis", Pearson, (2004)

Software: All computational examples will be presented using MATLAB. This software is available on the computers in the computer lab. Lab hours (see above) are reserved for unsupervised work with computer–based assignments. Unless they are reserved for large–class tutorials, students should be able to work in the computer labs in ABB also outside the allocated time–slots. Students are as well encouraged to purchase "The Student Edition of MATLAB" to be able to work with MATLAB at home. During the lectures some introduction to MATLAB will be provided.

Prerequisites: Advanced Calculus (MATH 2A03 or 2X03) and Numerical Algebra (MATH 2T03).

Assignments: Six home assignments will be handed out in class every second week, starting from January 17. Solutions of the assignments should be submitted via e-mail to the instructor using the template provided. The assignments are due by midnight on Wednesday the following week. Late submissions will not be accepted. Only five best assignments are counted towards the final mark. The assignments and solutions will be posted on the course webpage.

Test: There will be one in–class test on February 8. It will last 50 minutes and will cover analytical issues only (no programming). Only the McMaster standard calculator Casio fx-991 will be allowed during the quiz.

Final Exam: The course will be completed with a three–hour final examination. The date and location of the final exam will be announced by the Registrar's office in mid–term.

Marking scheme:

- Final exam (3 hrs) — 50%
- Test (50 min) — 10%
- Five homework assignments — 40%

Excused Absences: Exemptions from the assignments or tests for valid reasons are possible, but must be requested through the office of the Associate Dean of the Faculty that you are registered with. In the event of an exemption, no make up test or assignment will be administered, but your course grade will be re–weighted by increasing the weight of the final examination to compensate for the missed test or assignment.

Senate Policy Statement: The course is regulated by the following documents: *Statement on Academic Ethics* and *Senate Resolutions on Academic Dishonesty*. Any student who infringes one of these resolutions will be treated according to the published policy. In particular, academic dishonesty includes: (1) plagiarism, e.g. the submission of work that is not one's own, (2) improper collaboration in group work on home assignments, (3) copying or using unauthorized aids tests and examinations. It is your responsibility to understand what constitutes academic dishonesty, referring to *Academic Integrity Policy*.