TEST #1

13:30–14:20, February 8 (Thursday) in BSB/122

Make sure to put your name and ID number in the top–left corner of the answer sheet

No textbooks or notes allowed!

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1. State the Weierstrass Approximation Theorem. Compare the conditions and assertions made by this theorem with those made by the Taylor expansion theorem. *[2 points]*

2. Describe the fixed–point iteration technique for solution of nonlinear equations in the form $f(x) = 0$, where $f : \mathbb{R} \rightarrow \mathbb{R}$; in particular, derive the conditions under which it converges to a solution R and estimate the rate of convergence, i.e., the rate at which $|e_{n+1}| = |x_{n+1} - R|$ vanishes (the subscript n denotes the iteration count).*[2 points]*

3. You are given an algebraic system in the form $\mathbb{A}\mathbf{x} = \mathbf{b}$; derive an estimate of the solution error $\|\mathbf{x} - \overline{\mathbf{x}}\|$ when the system matrix is replaced with a perturbed matrix $\overline{\mathbb{A}} = \mathbb{A} + \mathbb{E}$ and the new system is $\overline{\mathbb{A}}\overline{\mathbf{x}} = \mathbf{b}$; comment how this error depends on the properties of the matrix $\mathbb{A}$. *[2 points]*

4. You are given a large number N of linear algebraic systems with the same matrix $\mathbb{A}$ and different right–hand side vectors $\mathbf{b}_1, \dots, \mathbf{b}_N$. Assuming that $\mathbb{A}$ is full, propose a computationally efficient way of solving these N systems. *[2 points]*