HOMEWORK #4

Due: November 26 (Thursday) by midnight

Instructions:

- The assignment consists of *three* questions, worth 4, 3 and 3 points.
- Submit your assignment *electronically* to the Email address `math3q03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH3Q03/template.m` (see also the link in the "Computer Programs" section of the course website); submissions non compliant with this template will not be accepted.
- All plots should have suitable axis labels and legends.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. We shall focus on Runge's function

$$f(x) = \frac{1}{1+25x^2}, \qquad x \in [-1,1]. \tag{1}$$

   (a) The expression for the interpolation error $E(x) = f(x) - p_n(x)$ involves an upper bound on the n-th derivative of the function $f(x)$, namely,

$$M_n := \max_{-1 \le x \le 1} |f^{(n)}(x)|.$$

   In `figure(1)` plot the absolute values of the derivatives of the function $f(x)$, i.e., $|f^{(n)}(x)|$ for $n = 1, \dots, 5$ as a function of x using logarithmic scaling for the vertical axis.

   (b) By plotting M_n versus n for $n = 1, \dots, 5$ using suitable scaling of the axes, estimate how M_n depends on n (i.e., is this dependence linear, quadratic, etc.?). The plot should appear as `figure(2)` and your answer should be printed on the screen.

   Perform these computations using `chebfun`.
   (4 points)

2. Consider a modified version of Runge's function

$$f_\alpha(x) = \frac{1}{1+\alpha^2 x^2}, \qquad x \in [-1,1], \tag{2}$$

   where $\alpha > 0$ is a parameter. In `figure(3)` plot the maximum errors of polynomial interpolation on a uniform (equispaced) grid $E_{\max}(n) := \max_{-1 \le x \le 1} |f_\alpha(x) - p_n(x)|$ as a function of the degree n of the interpolating polynomial for decreasing values of the parameter $\alpha = 5.0, 4.5, \dots, 0.5$ (in the figure there should be one data set for each value of α). Use logarithmic scaling for the vertical axis and the values $n = 2, 4, \dots, 50$ for the horizontal axis. These computations should be performed using `chebfun`.

In addition, answer the following questions (by printing brief messages on the screen):

(a) what effect does the parameter α have on the convergence of the polynomial interpolation?

(b) what are the origins of the errors observed in `figure(3)`?

(3 points)

3. Consider again the data $\{n, M_n\}$, $n = 1, \ldots, 5$, obtained in question 1b above. Find a *least-squares approximation* of this data using a suitable two-parameter fitting formula and print this formula together with its parameters on the screen. Plot this optimal fit using a dashed line and the original range of n in `figure(2)` and also print the total square error of the approximation on the screen.
(3 points)