MATH 3Q03
NUMERICAL EXPLORATIONS
(Fall 2015)

Time & Place: 9:30-10:20 on Mondays & Thursdays and 10:30-11:20 on Tuesdays in HH/217

Instructor: Dr. Bartosz Protas
    Office HH 326, Ext. 24116, Email: `bprotas@mcmaster.ca`
    Office Hours: 1:30-2:30pm on Tuesdays and 11:30-12:30 on Thursdays,
      or by appointment

Teaching Assistants: None

Course Webpage: available on http://ms.mcmaster.ca/bprotas/MATH3Q03/

Outline of the Course: In this course we will study key questions of numerical analysis such as approximation of functions and approximate differentiation and integration. We will see how various problems arising in calculus (both in single and in multiple variables) can be solved approximately, but with controlled accuracy, using computer algorithm. In addition to proving theorems about various numerical methods, we will develop, analyze and implement actual computational algorithms using MATLAB. We will also show how computational techniques can be used to illustrate and verify different results of mathematical analysis. As a highlight of the course, we will introduce `Chebfun` which is a MATLAB toolbox for performing hybrid numerical-symbolic computations with very high accuracy.

Topics: [the actual order may be different; characters in brackets represent the reference ("I" means that the material will be provided by the instructor)]

1. Introduction & Review of the Background Material
   - basic definitions [GP],
   - introduction to MATLAB and Chebfun [GP,T],
   - properties of polynomials [GP,T],
   - solution of systems of equations: linear & nonlinear [GP].

2. Interpolation
   - Vandermonde, Lagrange & Newton interpolation [GP],
   - error analysis: Runge phenomenon [GP, T, I],
   - Chebyshev interpolation [T],
   - trigonometric interpolation [GP].

3. Approximation
   - best approximations and orthogonal projections [T,I],
   - systems of orthogonal polynomials [T],
   - finding best approximations [GP,T].

4. Numerical Differentiation and Integration
   - derivatives via finite differences, error analysis [GP],
   - Richardson extrapolation [GP],
   - numerical quadratures, error analysis [GP],
   - spectral differentiation [T],
   - Gaussian quadratures [T].

5. Special Topics

- relation between interpolation and approximation [T],
- collocation vs. Galerkin methods for differential equations [I].

Primary Reference:

GP M. Grasselli and D. Pelinovsky, "Numerical Mathematics", Jones and Bartlett, (2008).

Secondary Reference:

T N. Trefethen, "Approximation Theory and Approximation Practice", SIAM, (2013) (we will focus primarily on the first chapters, six of which are available free of charge on the Author's webpage; all chapters can be generated from the source files provided)

Software: All computational examples will be presented using MATLAB and `Chebfun`. While MATLAB is available on the computers in most of the computer labs on the campus, students are encouraged to purchase *The Student Edition of MATLAB* to be able to work with MATLAB at home. `Chebfun` can be downloaded free of charge from http://www.chebfun.org/download/.

During the lectures we will provide introduction to `Chebfun`, however, students are expected to have already some familiarity with MATLAB.

Prerequisites: Advanced Calculus (MATH 2A03 or 2X03) and Introduction to Numerical Analysis (MATH 2T03).

Mid-Term Tests: There will be two in-class tests planned tentatively on October 8 and November 19. They will last 50 minutes and will cover analytical issues only (no programming). Only the McMaster standard calculator Casio fx-991 will be allowed during the tests.

Assignments: Five homework assignments will be posted on the course website on the dates indicated in the table below. The assignments will be due by midnight on Thursday the following week. Solutions of the assignments should be submitted by e-mail to `math3q03@math.mcmaster.ca` using the template provided. Late submissions will not be accepted. Only four best assignments are counted towards the final mark. The assignments and solutions will be posted on the course webpage.

Homework Post & Due Dates (tentative):

All assignments are due by midnight on Thursdays as indicated in the table below.

#	Post Date	Due Date
HW #1	September 24	October 1
HW #2	October 22	October 29
HW #3	November 5	November 12
HW #4	November 19	November 26
HW #5	December 3	December 10

Final Exam: The course will be completed by a 2.5 hour final examination. The date and location of the final exam will be announced by the Registrar's office in mid–term. The exam will cover all course material.

Marking scheme: The final mark will be computed using the following scheme:

Two mid-term tests	20% (10% each)
Final Examination	40%
Four best homework assignments	40% (10% each)

The instructor reserves the right to alter the grade in justified cases. In such situations, however, the grade can only be increased.

Relief for Missed Work: In the event of an absence for medical or other reasons, students should review and follow the Academic Regulation in the Undergraduate Calendar Requests for Relief for Missed Academic Term Work. Please note these regulations have changed beginning Fall 2015.

Academic Ethics: Academic dishonesty consists of misrepresentation by deception or by other fraudulent means and can result in serious consequences, e.g., the grade of zero on an assignment, loss of credit with a notation on the transcript (notation reads: "Grade of F assigned for academic dishonesty"), and/or suspension or expulsion from the University. It is your responsibility to understand what constitutes academic dishonesty. For information on the various kinds of a academic dishonesty please refer to the Academic Integrity Policy (https://www.mcmaster.ca/academicintegrity/) specifically Appendix 3. The following illustrates only three forms of academic dishonesty: 1. Plagiarism, e.g., the submission of work that is not one's own or which has been used for other credit. 2. Improper collaboration in group work. 3. Copying or using unauthorized aids in tests and exams.

Course Modifications: The instructor and University reserve the right to modify elements of the course during the term. The University may change the dates and deadlines for any or all courses in extreme circumstances. If either type of modification becomes necessary, reasonable notice and communication with the students will be given with explanation and the opportunity to comment on changes. It is the responsibility of the student to check their McMaster email and course websites weekly during the term and to note any changes.