

TEST #1

9:30–10:20am, October 8 (Thursday), 50 minutes, 10 points max
(no textbooks, no notes)

Write your name and student number on the top of this sheet.
Write your answers on the reverse side and/or attach additional sheets as
necessary.

1. The following instructions are executed in **MATLAB**:

```
a = 10.0;  
b = 1.0e-20;  
a = a + b
```

What will be the *exact* value of the variable **a**? Justify your answer.

[2 points]

2. Explain what *linear convergence* and *quadratic convergence* mean in the context of root-finding methods for problems of the type $f(x) = 0$, where $f : \mathbb{R} \rightarrow \mathbb{R}$ is a given function. Provide an example of a method characterized by each of these two types of convergence.

[2 points]

3. Describe the fixed-point iteration technique for solution of nonlinear equations in the form $f(x) = 0$, where $f : \mathbb{R} \rightarrow \mathbb{R}$; in particular, derive the conditions under which it converges to a solution R and estimate the rate of convergence, i.e., the rate at which $|e_n| = |x_n - R|$ vanishes (x_n denotes the approximation obtained at the n -th iteration).

[2 points]

4. You are solving the equation $f(x) = 0$ with some function $f : \mathbb{R} \rightarrow \mathbb{R}$ using the *bisection* method. Knowing that the root R of this equation belongs to the interval $[0, 1]$, how many iterations does one have to perform in order to ensure that the error $|e_n| = |R - x_n|$ is not greater than 10^{-3} (x_n denotes the approximation of the root obtained at the n -th iteration)?

[2 points]

5. You are given the following two systems of linear equations

$$(a) \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (b) \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

How many solutions $[x_1 \ x_2]^T$ does each of these systems admit? Justify your answer.

[2 points]