## USEFUL FORMULAS

- Lagrange interpolating polynomial and functions based on $\{x_i, y_i\}_{i=1}^n$:

$$p_n(x) = \sum_{k=0}^{n} y_k \Phi_k(x), \text{ where } \Phi_k(x) = \frac{\Pi_{i=0,i\neq k}^n (x - x_i)}{\Pi_{i=0,i\neq k}^n (x_k - x_i)}, \quad k = 0, \dots, n,$$

- Error of polynomial interpolation based on $\{x_k, y_k\}_{k=1}^n$:

$$E = f(x) - p_n(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} \prod_{k=0}^{n} (x - x_k)$$

- Chebyshev polynomials:

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n \geq 1, \ x \in [-1, 1]$$

- Chebyshev polynomials aliased to $T_m(x)$ ($0 \leq m \leq n$) on a Chebyshev grid with $n+1$ points:

$$T_m, \ T_{2n-m}, \ T_{2n+m}, \ T_{4n-m}, \ T_{4n+m}, \ T_{6n-m}, \dots$$

- Suppose that the function $f$ defined on $[-1, 1]$ has $(\nu - 1)$ absolutely continuous derivatives and its $\nu$-th derivative is of bounded variation $V$ ($\nu \geq 1$). Then, the errors of Chebyshev projection $f_n$ and Chebyshev interpolant $p_n$ are subject to the following bounds valid for $n > \nu$

$$\|f - f_n\|_\infty \leq \frac{2V}{\pi \, \nu \, (n - \nu)^\nu},$$
$$\|f - p_n\|_\infty \leq \frac{4V}{\pi \, \nu \, (n - \nu)^\nu}.$$

**THE END**