HOMEWORK #4

Due: March 23 (Thursday) by midnight

Instructions:

- The assignment consists of *three* questions, worth 5, 3 and 2 points.
- Submit your assignment *electronically* to the Email address `math3q03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/bprotas/MATH3Q03/template.m` (see also the link in the "Computer Programs" section of the course website); submissions non compliant with this template will not be accepted.
- All plots should have suitable axis labels and legends.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

---

1. We shall again focus on Runge's function

$$f(x) = \frac{1}{1+25x^2}, \qquad x \in [-1,1]. \tag{1}$$

(a) The expression for the interpolation error $E(x) = f(x) - p_n(x)$ involves an upper bound on the n-th derivative of the function $f(x)$, namely,

$$M_n := \max_{-1 \le x \le 1} |f^{(n)}(x)|.$$

In `figure(1)` plot the absolute values of the derivatives of the function $f(x)$, i.e., $|f^{(n)}(x)|$ for $n = 1, \dots, 5$ as a function of x using logarithmic scaling for the vertical axis.

(b) By plotting M_n versus n for $n = 1, \dots, 5$ using suitable scaling of the axes, estimate how M_n depends on n (i.e., is this dependence linear, quadratic, etc.?). The plot should appear as `figure(2)` and your answer should be printed on the screen.

(c) Find a *least-squares approximation* of the data $\{n, M_n\}$, $n = 1, \dots, 5$ data using a suitable two-parameter fitting formula and print this formula together with its parameters on the screen. Plot this optimal fit using a dashed line and the original range of n in `figure(2)` and also print the total square error of the approximation on the screen.

Perform these computations using `chebfun`.
(5 points)

2. You are given the function

$$f(x) = e^x \sin(\pi x), \qquad x \in [-1, 1].$$

Construct a family of truncated Chebyshev series expansions $f_n(x) = \sum_{k=0}^{n} a_k T_k(x)$ of $f(x)$, where T_k are the Chebyshev polynomials, a_k the corresponding Chebyshev coefficients and $n = 0, 1, \ldots, 20$. Plot $f(x)$ and $f_n(x)$, $n = 0, 1, \ldots, 20$, as functions of x using, respectively, thick and thin lines in `figure(3)`. Then, plot the magnitudes of the Chebyshev coefficients $|a_k|$ as function of $k = 0, 1, \ldots, 20$. This plot should appear as `figure(4)` and use the logarithmic scaling of the vertical axis. If `f` is a `chebfun` object representing the function $f(x)$, the Chebyshev coefficients can be obtained as follows (the second command is required to put them in the right order)

```
ak = chebpoly(f), ak = ak(end:-1:1);
```

(3 points)

3. Suppose $n = 10$ and construct Chebyshev grid with $(n + 1)$ points. Compute the number

$$S = (\ [\text{sum of the last 5 digits in your student number}] \mod n\ ) \tag{2}$$

and plot the Chebyshev polynomial $T_S(x)$ defined on $[-1, 1]$ together with 6 lowest-degree polynomials $T_m(x)$ which are aliased to $T_S(x)$. This plot should appear as `figure(5)` with the polynomial $T_S(x)$ marked with a thick solid line and the aliases $T_m(x)$ marked with thin solid lines. Mark the intersection points of the graphs of $T_S(x)$ and $T_m(x)$ with small circles. To reveal small-scale features of the aliases, in `figure(6)` show the same data in a region of width $\Delta x = 0.05$ centered around the leftmost Chebyshev point x_i located in the *interior* of the domain $[-1, 1]$. Perform these computations using `chebfun`.
(2 points)