TEST #1
1:30–2:20pm, February 2 (Thursday), 50 minutes, 10 points max
(no textbooks, no notes)

Write your name and student number on the top of this sheet.
Write your answers on the reverse side and/or attach additional sheets as
necessary.

---

1. You evaluate the function
$$f(x) = \left(\sqrt{x}\right)^2$$
using a computer, for example with `MATLAB`. Comment on the accuracy of this operation when

   (a) $x = 0$,
   (b) $x = 2$,
   (c) $x = 4$.

   Justify your answer.
   [2 points]

2. Explain what *linear convergence* and *quadratic convergence* mean in the context of root-finding methods for problems of the type $f(x) = 0$, where $f : \mathbb{R} \to \mathbb{R}$ is a given function. Provide an example of a method characterized by each of these two types of convergence.
   [2 points]

3. Describe Newton's method for solution of nonlinear equations in the form $f(x) = 0$, where $f : \mathbb{R} \to \mathbb{R}$, and derive the conditions under which it converges to a root R. What happens when the derivative of $f(x)$ cannot be evaluated analytically?
   [2 points]

4. You are solving the equation $f(x) = 0$ with some function $f : \mathbb{R} \to \mathbb{R}$ using the *bisection* method. Knowing that the root R of this equation belongs to the interval $[0, 1]$, what will be the largest possible error $|e_n| = |R - x_n|$ after $n = 5$ iterations?
   [2 points]

5. You are solving the equation $f(x) = 0$ numerically, where $f(x) = (x-1)^2 \, e^{-x^2}$. What can we say about the convergence of iterations when

   (a) the bisection method is used with the initial approximations chosen as $x_0 = 0$ and $x_1 = 2$, and

   (b) Newton's method is used with the initial approximation chosen as $x_0 = \frac{1}{2}\left(\sqrt{5} + 1\right)$?

   Justify your answer.
   [2 points]