TEST #2

9:30–10:20am, November 19 (Thursday), 50 minutes, 10 points max
(no textbooks, no notes)

Write your name and student number on the top of this sheet.
Write your answers on the reverse side and/or attach additional sheets as necessary.

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1. Derive Newton's method for the solution of the problem $\mathbf{F}(\mathbf{x}) = \mathbf{0}$, where $\mathbf{F} : \mathbb{R}^n \to \mathbb{R}^n$, and describe the main steps which have to be performed at every iteration. What are the properties required of the function $\mathbf{F}$ for this approach to be applicable?
[2 points]

2. Your are given the following functions

   (a) $f(x) = e^{-x^2}, \quad x \in \mathbb{R}$,
   (b) $f(x) = \sin^2(2x), \quad x \in [0, 2\pi]$,
   (c) $f(x) = \begin{cases} -1, & x \in [-1, 0) \\ 1, & x \in [0, 1] \end{cases}$,
   (d) $f(x) = |x|, \quad x \in [-1, 1]$.

   According to the Weierstrass Approximation Theorem, which of these functions can be approximated on its domain of definition to an arbitrary precision with a polynomial? In each case justify your answer!
[2 points]

3. You are given the following data points: $(x_0, y_0) = (-1, 0)$, $(x_1, y_1) = (0, -1)$ and $(x_2, y_2) = (1, 2)$. Use Lagrange functions to construct an interpolating polynomial $p_n(x)$, such that $p_n(x_i) = y_i$, $i = 0, 1, 2$. The Lagrange functions are given by

$$\Phi_k(x) = \frac{\Pi_{i=0, i\neq k}^n (x - x_i)}{\Pi_{i=0, i\neq k}^n (x_k - x_i)}, \quad k = 0, \dots, n,$$

   and the corresponding interpolating polynomial is $p_n(x) = \sum_{k=0}^n y_k \Phi_k(x)$. You need not use the barycentric interpolation formula.
[4 points]

4. What can we say about the convergence of polynomial interpolation (i.e., the behavior of the errors as the degree n of the interpolating polynomial increases) when the following two functions are interpolated on a *uniform (equispaced) grid*

   (a) $f(x) = e^x, \quad x \in [-1, 1]$,
   (b) $f(x) = e^{|x|}, \quad x \in [-1, 1]$?

[2 points]