MATH 4Q03/6Q03
NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS
(Winter 2005)

Time & Place:

- Lectures — Wednesday, 19:00-22:00 in HH/109
- Computer Labs — Tuesday 16:30-17:20 in BSB/241, Wednesday 14:30-15:20 in BSB/245, Wednesday 17:30-18:20 in BSB/245

Instructor: Dr. Bartosz Protas
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Course Webpage: http://www.math.mcmaster.ca/~bprotas/MATH4Q03a

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Outline of the Course: This course will provide an overview of classical solution methods for ordinary and partial differential equations. The focus will be on finite–difference techniques, but time permitting, certain more advanced approaches such as finite element and spectral methods will also be presented. At the beginning we will also review interpolation and approximation techniques as well as methods of numerical differentiation and integration. As an illustration of the presented algorithms we will use simple MATLAB codes to solve certain computational problems commonly arising in Physics and Engineering.

Topics: (numbers in **bold** refer to sections of the textbook by Gerald & Wheatley, whereas numbers in *italic* refer to sections of the textbook by S. S. Rao.)

1. Introduction & Review
   - Types of errors in numerical computation **(0.4-0.5)** *(1.4,1.6)*
   - Root finding methods for nonlinear equations **(1.1-1.6)** *(2.3-2.9)*
   - Numerical linear algebra **(2.1-2.5)** *(3.3-3.14)*
2. Interpolation & Approximation
   - Polynomial interpolation **(3.1, 3.2)** *(5.3,5.5,5.6)*
   - Spline interpolation **(3.3, 3.4)** *(5.8)*
   - Least squares approximations **(3.6)** *(5.9)*
3. Numerical differentiation and integration
   - Forward, backward and central difference approximation of derivatives **(5.1)** *(7.4-7.8)*
   - Newton–Cotes integration formulas **(5.2,5.3,5.6)** *(8.3,8.4,8.7)*
   - Gaussian integration formulas **(5.6)** *(8.8)*
4. Ordinary differential equations: initial–value problems
   - Single–step Euler and Runge–Kutta methods **(6.1,6.2,6.3)** *(9.5,9.6,9.7)*
   - Multi–step explicit and implicit Adams methods **(6.4,6.6)** *(9.9,9.10,9.12)*
5. Ordinary differential equations: boundary–value problems
   - Shooting methods **(6.7)** *(10.3,10.4)*

- Global finite-difference methods **(6.7)** *(10.5,10.6)*

6. Finite–difference methods for partial differential equations
   - Iterative methods for the Laplace equation **(8.1)** *(11.4)*
   - Explicit and implicit methods for the heat equation **(8.2)** *(11.5,11.6,11.8)*
   - Explicit and implicit methods for the wave equation **(8.3)** *(11.9)*
7. Advanced numerical methods
   - Finite–element methods **(9.2,9.3)** *(13.4,13.5,13.7)*
   - Spectral methods (material will be provided by the instructor)

Primary Reference:

1. C. F. Gerald & P. O. Wheatley, "Applied numerical analysis", Pearson, (2004)

Secondary References:

2. S. S. Rao, "Applied numerical methods for engineers and scientists", Prentice Hall, (2002).
3. R. J. Schilling & S. L. Harris, "Applied numerical methods for engineers using MATLAB and C", Brooks/Cole, (2000).

Software: All of the computational examples will be presented using MATLAB. This software is available on the computers in the computer lab. Lab hours (see above) are reserved for unsupervised work with computer–based assignments. Unless they are reserved for large–class tutorials, students should be able to work in the computer labs in BSB also outside the allocated time–slots. Students are as well encouraged to purchase "The Student Edition of MATLAB" to be able to work with MATLAB at home. While a limited introduction to MATLAB will be provided, it is recommended that students grasp basic MATLAB programming skills working either in the computer labs or with the personal student edition.

Custom Courseware:

- D. E. Pelinovsky, "Scientific Computing with MATLAB" McMaster, (2004) (in due time various elements of this package will be linked through the course webpage)

Prerequisites: Calculus, Ordinary and Partial Differential Equations, Numerical Algebra, Basic programming skills in MATLAB

Assignments: Six home assignments will be handed out in class every second week, starting from January 12. Solutions of the assignments should be submitted by e-mail to `math4q03@math.mcmaster.ca`. The assignments are due by midnight on Wednesday the following week. Only five best assignments are counted towards the final mark. The assignments and solutions will be posted on the course webpage.

Class Quiz: There will be one in–class quiz on February 9. It will last 50 minutes and will cover analytical issues only (no programming). Only the McMaster standard calculator Casio fx-991 will be allowed during the quiz.

Final Exam: The course will be completed by a three–hour final examination. The date and location of the final exam will be announced by the Registrar's office in mid–term.

Marking scheme:

- Final exam (3 hrs) —50%
- Test (50 min) —10%
- Five homework assignments —40%

Senate Policy Statement: The course is regulated by the following documents: *Statement on Academic Ethics* and *Senate Resolutions on Academic Dishonesty*. Any student who infringes one of these resolutions will be treated according to the published policy. In particular, academic dishonesty includes: (1) plagiarism, e.g. the submission of work that is not one's own, (2) improper collaboration in group work on home assignments, (3) copying or using unauthorized aids tests and examinations. It is your responsibility to understand what constitutes academic dishonesty, referring to *Academic Integrity Policy*.