HOMEWORK #2

Due: February 8 (Thursday) by midnight

Instructions:

- The assignment consists of *two* questions, worth 4 points each.
- Submit your assignment *electronically* (via Email) to the address `math4q03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH4Q03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. You are given five algebraic systems $\mathbb{A}\mathbf{x}_1 = \mathbf{y}_1$, $\mathbb{A}\mathbf{x}_2 = \mathbf{y}_2$, $\mathbb{A}\mathbf{x}_3 = \mathbf{y}_3$, $\mathbb{A}\mathbf{x}_4 = \mathbf{y}_4$, and $\mathbb{A}\mathbf{x}_5 = \mathbf{y}_5$, with the same matrix

$$\mathbb{A} = \begin{bmatrix} 6 & 1 & 0 & 3 \\ 5 & 8 & 4 & 0 \\ 2 & 2 & 7 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix} \tag{1}$$

and different right–hand side (RHS) vectors, such that $\mathbf{y}_1 = [4\ \ 1\ \ 9\ \ 2]^T$ and the remaining RHS vectors are defied as $[\mathbf{y}_2]_k = ([\mathbf{y}_1]_k)^2$, $[\mathbf{y}_3]_k = ([\mathbf{y}_1]_k)^3$, $[\mathbf{y}_4]_k = ([\mathbf{y}_1]_k)^{-1}$ and $[\mathbf{y}_5]_k = ([\mathbf{y}_1]_k)^{1/2}$, where $k = 1, \dots, 4$.

   (a) using MATLAB function `lu` perform LU decomposition of the matrix $\mathbb{A}$; print out the resulting matrices $\mathbb{L}$ and $\mathbb{U}$ (note that, due to the specific structure of the matrix $\mathbb{A}$, the matrix $\mathbb{L}$ returned by `lu` should indeed be lower triangular, rather than "psychologically lower triangular" as mentioned by `help lu`),

   (b) Write your own two functions `Lbck` and `Ubck` that solve an algebraic system with, respectively, lower and upper triangular matrix using back substitution; use these functions to solve the above five system in two steps, i.e., first solve $\mathbb{L}\mathbf{z}_i = \mathbf{y}_i$ and then $\mathbb{U}\mathbf{x}_i = \mathbf{z}_i$, $i = 1, \dots, 5$ (do not use the operator "\"!),

   (c) print out in a single row the first elements of the five solutions vectors $\mathbf{x}_1, \dots, \mathbf{x}_5$.

   (4 points)

2. Your are given the function $f(x) = x\cos(x^2)$ in the interval $\Omega = [-\pi, \pi]$ which is discretized using a uniformly distributed grid with $N+1$ grid points (take $x_0 = -\pi$ and $x_N = \pi$).

   (a) consider the Vandermonde approach to interpolation and determine the value of N for which the condition number κ (obtained using MATLAB function `cond`) of the interpolation matrix exceeds the threshold $E = 10^6$; write out this value of N,

   (b) calculate the Lagrange interpolating polynomials in the case when $N = 4$ and plot them using different colors and the step size $h = x_{i+1} - x_i = \frac{\pi}{50}$,

(c) using two separate figures plot the function $f(x)$ in the interval Ω together with the interpolating polynomials constructed using the Lagrange polynomials with $N=4$ and $N=16$; use the same h as before,

(d) using the Lagrange interpolating polynomials constructed above determine and print out the interpolation error at $x=0.01$ and with $N=2,4,\dots,10$; repeat this using the function $g(x)=|f(x)|$; why do the errors decrease with N in one case, but not in the other?

(4 points)