HOMEWORK #3

Due: March 5 (Monday) by midnight

Instructions:

- The assignment consists of *two* questions, worth 4 points each.
- Submit your assignment *electronically* (via Email) to the address `math4q03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH4Q03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. You are given the following set of data points Construct *cubic* splines interpolating these data

x_i	-1	-0.75	-0.5	-0.25	0.0	0.25	0.5	0.75	1
y_i	0	0	0	-1	-0.5	-1	0	0	0

point making the following assumptions as regards the two additional conditions required to close the system:

(a) $s_0 = s_n = 0$ ("natural" splines),
(b) slopes at the endpoints are given by $f'(x_0) = 3$ and $f'(x_n) = -3$,
(c) the end cubics approach parabolas at their extremities,
(d) the values of s_0 and s_n are extrapolated linearly from the values at the neighboring nodes, i.e., s_1, s_2 and s_{n-2}, s_{n-2}, respectively.

Plot the four interpolants together with the original data on a single plot using different colors (make sure to provide a legend!).
HINT — see the four boxed points on page 172 in the textbook for relevant formulas; you may start by modifying the code `spline_01.m` posted on the course webpage.
(4 points)

2. You are given the *cardinal Whittaker* function $f(x) = \frac{\sin(x)}{x}$ in the interval $\Omega = [-\pi, \pi]$.

(a) Using the step size $h = 0.2$ calculate approximations to $f'(x)$ in Ω using the following methods:

i. second–order *one–sided* (forward) differences
ii. second–order central differences
iii. complex step derivative[1]

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[1] Assume that $F(z)$ is a complex analytic extension of $f(x)$, i.e., $F : \mathbb{C} \to \mathbb{C}$ and $F(x) = f(x)$ for $x \in \mathbb{R}$. Calculate the Taylor series expansion $F(x+ih) = F(x) + ih\frac{\partial F}{\partial y} - \frac{h^2}{2}\frac{\partial^2 F}{\partial y^2} + O(h^3)$. Using a property of analytic functions known as the Cauchy–Riemann condition, i.e., $\frac{\partial F}{\partial y} = \frac{\partial F}{\partial x} = f'(x)$, rearranging, and taking the imaginary part we obtain the following approximation of the derivative: $f'(x) = \frac{\Im(F(x+ih))}{h} + O(h^3)$, known as the *complex step derivative*.

iv. fourth–order central differences

Plot the resulting curves using solid lines in different colors and the derivative computed analytically using a dashed line.

(b) Using the four approximate methods mentioned above, calculate for $x_0 = \frac{\pi}{12}$ the relative errors of the derivatives as $\left|\frac{f'_{approx}(x_0) - f'_{exact}(x_0)}{f'_{exact}(x_0)}\right|$ and plot them on a log–log graph as a function of the step size h (for h use the values obtained with `logspace(-10,0,10)`).

(4 points)