HOMEWORK #4

Due: March 19 (Monday) by midnight

Instructions:

- The assignment consists of *two* questions, worth 4 points each.
- Submit your assignment *electronically* (via Email) to the address `math4q03@math.mcmaster.ca`; hardcopy submissions will not be accepted.
- It is obligatory to use the MATLAB template file available at `http://www.math.mcmaster.ca/~bprotas/MATH4Q03/template.m` (see also the link in the "Computer Programs" section of the course website on the left); submissions non compliant with this template will not be accepted.
- Make sure to enter your name and student I.D. number in the appropriate section of the template.
- Late submissions and submissions which do not comply with these guidelines will not be accepted.

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1. You are given the following integral $\int_2^4 (x^8 + x^3 - x + 2)\,dx$. Evaluate this integral numerically using Simpson's "1/3" rule and the Gaussian quadrature subdividing the interval $[a,b]$ into $2,4,\ldots,100$ subintervals. Then

   (a) determine the *relative* errors obtained with the two methods and plot them as a function of the inverse of the number of points used to evaluate the quadratures; use the log–log coordinates;

   (b) using a least–squares fit, determine empirically the order of accuracy of Simpson's "1/3" rule,

   (c) explain why the error in Gaussian integration stops to decrease at some point.

   HINT — Use the function `lgwt` posted on the course website to determine the quadrature points and weights for the Gaussian integration.
   (4 points)

2. Consider the following nonlinear Initial Value Problem:

$$\frac{d^2y}{dt^2} + y^2 = t \qquad\qquad t \in [0,10],$$
$$y(0) = 1,\ \frac{dy}{dt}(0) = 0.$$

   Using the time step $\Delta t = 2 \cdot 10^{-2}$ solve this equation employing the following approximate methods:

   (a) explicit Euler,

   (b) Adams–Bashforth (use the explicit Euler method as a stater at the first time step),

   (c) 4^{th}–order Runge–Kutta.

   Plot the results obtained using each method as *phase diagrams*, i.e., in the form $\frac{dy}{dt}(t) = f(y(t))$ (the time t serves as a parameter), on a single plot using different colors and line types.
   (4 points)