TEST #1

9:30–10:20, February 15 (Thursday) in REF/102

Make sure to put your name and ID number in the top–left corner of the answer sheet

No textbooks or notes allowed!

---

1. You are given an equation $f(x) = 0$, where $f : \mathbb{R} \to \mathbb{R}$. Consider a modified version of this equation $\alpha f(x) = 0$, where $\alpha \neq 0$ is a real number. Construct a fixed-point algorithm for solution of the modified equation. How to choose the parameter α at every iteration, so that this algorithm will be identical with Newton's method applied to the original equation?
   [2 points]

2. You are given an algebraic system in the form $\mathbb{A}\mathbf{x} = \mathbf{b}$; derive an estimate of the relative solution error $\frac{\|\mathbf{x}-\overline{\mathbf{x}}\|}{\|\mathbf{x}\|}$ when the system matrix is replaced with a perturbed matrix $\overline{\mathbb{A}} = \mathbb{A} + \mathbb{E}$ and the new system is $\overline{\mathbb{A}}\overline{\mathbf{x}} = \mathbf{b}$; comment how this error depends on the properties of the matrix $\mathbb{A}$.
   [2 points]

3. You are given a large number N of linear algebraic systems with the same matrix $\mathbb{A}$ and different right–hand side vectors $\mathbf{b}_1, \dots, \mathbf{b}_N$. Assuming that $\mathbb{A}$ is full, propose a cost–efficient way of solving these N systems (note that the optimal approach is not iterative).
   [2 points]

4. You are given two data points $\{x_1, y_1\}$ and $\{x_2, y_2\}$. Construct an interpolating polynomial $P_1(x)$ that will go through these two point using
   (a) the direct (Vandermonde) approach; (only derive, but do not solve, the algebraic system),
   (b) Lagrange polynomials,
   (c) divided differences.

   [2 points]

5. Given a set of data points $\{x_i, y_i\}_{i=1}^{N}$, where N is a large number, show how to solve the least–squares approximation problem using $y(x) = ae^{bx}$ as the interpolating function ($a, b \in \mathbb{R}$).
   [2 points]