MATH 741 — APPLIED MATHEMATICS I

Time: 11:00am–12:30pm on Tuesdays and Fridays (on-line)

Instructor: Dr. Bartosz Protas
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Office Hours: TBA
Course Webpage: http://www.math.mcmaster.ca/~bprotas/MATH741

Format of the Course: The course will be virtual in the synchronous format. All lectures will take place on-line via `Zoom` at the times indicated above. Students should be aware that, when they access the electronic components of a course using this platform, private information such as first and last names, user names for the McMaster e-mail accounts, and program affiliation may become apparent to all other students in the same course. The available information is dependent on the technology used. Continuation in a course that uses on-line elements will be deemed consent to this disclosure. If you have any questions or concerns about such disclosure please discuss this with the course instructor.

Outline of the Course: The main goal of this course is to offer an introduction to classical methods of applied mathematics. We will focus on the qualitative theory of systems of ordinary differential equations (ODEs). Following a review of standard results concerning existence and uniqueness of solutions and their continuous dependence on parameters, we will study linear system, stability theory, invariant manifolds, ending with a survey of periodic and homoclinic solutions. A second objective of this course is to introduce students to modern methods of symbolic and numerical computing useful in quantitative analysis. We will use the software environment MAPLE to illustrate a number of problems discussed in the course. In the optimistic variant, the specific topics to be discussed will include (number is parentheses correspond to sections in the textbook by L. Perko):

1. Elements of the ODE Theory
   (a) existence of solutions (2.1, 2.2),
   (b) uniqueness of solutions (2.2)
   (c) dependence on parameters (2.3)
   (d) flows defined by differential equations (2.5)
2. Linear Systems and Stability
   (a) properties of linear systems (1.3, 1.4)
   (b) solutions with homogeneous systems with constant coefficients (1.6, 1.7, 1.8)
   (c) critical points and linearized stability (1.9, 2.6)
   (d) Lyapunov functions and nonlinear stability (2.9)
3. Hyperbolic Theory
   (a) stable and unstable manifolds of dynamical systems (2.7, 2.10)
   (b) linearization of hyperbolic systems (2.8)
   (c) center manifold and nonlinear stability (2.11, 2.12)
   (d) normal forms (2.3)
4. Periodic and Homoclinic Orbits
   (a) Floquet theory and stability of periodic solutions (3.3)
   (b) Poincaré maps (3.4, 3.5)
   (c) Poincaré–Bendixon theory (3.6, 3.7, 3.8)
   (d) index theory and separatrix orbits (3.12)
   (e) structural stability (4.1)

Primary Reference:

1. L. Perko, *Differential Equations and Dynamical Systems*, Third Edition, Springer, (2008), ISBN 0387951164

Supplemental Reference:

2. S. Lynch, *Dynamical Systems with Applications Using MAPLE*, Second Edition, Birkhäuser, (2010). ISBN 978–0–8176–4389–8
3. R. K. Miller and A. N. Michel, *Ordinary Differential Equations*, Academic Press, (1982). ISBN 0–12–497280–2

In addition to the above references, example MAPLE codes will be made available to students on the course webpage.

Prerequisites: Real analysis and basic differential equations; no programming skills in MAPLE are required

Homework Assignments: There will be four homework assignment which may involve some elements of MAPLE programming. The *tentative* post and due dates are indicated in the table below. Submissions are due electronically at 11:59pm on the due date.

#	Post Date	Due Date
HW1	Wednesday, September 30	Wednesday, October 7
HW2	Wednesday, October 21	Wednesday, October 28
HW3	Wednesday, November 11	Wednesday, November 18
HW4	Wednesday, November 25	Wednesday, December 2

Grades: The final grades will be based on:

- four homework assignments ($4 \times 15\% = 60\%$),
- take–home final exam (40%).

The instructor reserves the right to alter your final grade, in which case, however, the grade may only be increased.

Senate Policy Statement: The course is regulated by the following documents: *Statement on Academic Ethics* and *Senate Resolutions on Academic Dishonesty*. Any student who infringes one of these resolutions will be treated according to the published policy. In particular, academic dishonesty includes: (1) plagiarism, e.g. the submission of work that is not one's own, (2) improper collaboration in group work on home assignments, (3) copying or using unauthorized aids tests and examinations. It is your responsibility to understand what constitutes academic dishonesty, referring to *Academic Integrity Policy*.

Important Notice: The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If either type of modification becomes necessary, reasonable notice and communication with the students will be given with explanation and the opportunity to comment on changes. It is the responsibility of the student to check their McMaster email and course websites weekly during the term and to note any changes.