McMaster University | SCIENCE
Department of Mathematics & Statistics

We recognize and acknowledge that McMaster University meets and learns on the traditional territories of the Mississauga and Haudenosaunee nations, and within the lands protected by the "Dish With One Spoon" wampum, an agreement amongst all allied Nations to peaceably share and care for the resources around the Great Lakes.

MATH 745 – Topics in Numerical Analysis
2023 Fall Term

Instructor: Dr. Bartosz Protas | **E-mail:** bprotas@mcmaster.ca | **Office:** HH-218/E |

Office Hours: 2:30–3:30pm on Tuesdays and Thursdays

Class Time and Location

- **Lectures:** 3:30–5:00pm on Tuesdays and Thursdays during the period from September 5 through December 6, with the exception of the week of October 9 (mid-term recess). See MOSAIC for classroom location.

Course Website

- https://math.mcmaster.ca/bprotas/MATH745/

Course Description

The course will focus on techniques for numerical solution of Partial Differential Equations (PDEs). The objectives of the course are essentially twofold: first, provide students with an understanding of the deeper mathematical foundations for certain classical numerical methods which they should already be familiar with, and, secondly, introduce students to more advanced numerical methods for PDEs. The course will address both theoretical aspects, such as error and stability analysis, as well as main implementation issues. The presented methods will be illustrated using well-known PDEs from mathematical physics.

Prerequisite(s): Numerical Analysis at the undergraduate level (including numerical methods for ODEs and PDEs), Partial Differential Equations, basic programming skills in MATLAB

Required Materials & Fees

1. Lecture notes and computer codes provided by the instructor on the course webpage
2. **Primary Reference:** L. N. Trefethen, *Spectral Methods in Matlab*, SIAM, (2000).
3. **Secondary Reference:** K. Atkinson and W. Han, *Theoretical Numerical Analysis: A Functional Analysis Framework*, Springer (TAM 39), (2001)
4. **Secondary Reference:** J. P. Boyd, *Chebyshev and Fourier Spectral Methods, Second Edition (Revised)*, Dover, (2001).
5. **Software:** MATLAB

Virtual Course Delivery

To follow and participate in virtual classes it is expected that you have reliable access to the following:

- A computer that meets performance requirements found here.
- An internet connection that is fast enough to stream video.
- Computer accessories that enable class participation, such as a microphone, speakers and webcam when needed.

If you think that you will not be able to meet these requirements, please contact uts@mcmaster.ca as soon as you can. Please visit the Technology Resources for Students page for detailed requirements. If you use assistive technology or believe that our platforms might be a barrier to participating, please contact Student Accessibility Services, sas@mcmaster.ca, for support.

Course Overview and Assessment

A tentative list of topics to be discussed:

1) Critical Review of Finite--Difference Methods
   a) Discretization of differential operators; incorporation of boundary conditions
   b) Accuracy and conditioning of numerical differentiation
   c) Advanced numerical differentiation (complex step derivative, Pade schemes, compact finite differences)

2) Review of Approximation Theory
   a) Functional analysis background (Hilbert spaces, inner products, orthogonality and orthogonal systems)
   b) Best approximations
   c) Interpolation theory

3) Spectral methods for PDEs
   a) Differentiation in spectral space
   b) Fourier and Chebyshev methods; fast transforms (FFT)
   c) Application to nonlinear problems (pseudo--spectral methods, dealiasing)

4) Multiresolution methods for PDEs
   a) Orthogonal wavelets
   b) Discrete wavelet transform (DWT)
   c) Multiresolution representation of functions

Evaluation

There will be two homework assignments which will involve elements of MATLAB programming. The *tentative* post and due dates are indicated in the table below. Submissions are due electronically at 11:59pm on the due date.

Assignment	Post Date	Due Date
HW#1	October 17	October 24
HW#2	November 14	November 21

There will also be two short quizzes during the lectures on October 19 and November 23 in addition to a final take-home project at the end of the course (details and topics for the project will be provided in the middle of the term).

The final grades will be computed as follows.

Grade Component	Weight
two quizzes	20% = 2 x 10%
two homework assignments	20% = 2 x 10%
final project	60%

Requests for Relief for Missed Academic Term Work

McMaster Student Absence Form (MSAF): In the event of an absence for medical or other reasons, students should review and follow the Academic Regulation in the Undergraduate Calendar "Requests for Relief for Missed Academic Term Work".

Academic Accommodation of Students with Disabilities

Students with disabilities who require academic accommodation must contact Student Accessibility Services (SAS) at 905-525-9140 ext. 28652 or sas@mcmaster.ca to make arrangements with a Program Coordinator. For further information, consult McMaster University's *Academic Accommodation of Students with Disabilities* policy.

Academic Accommodation for Religious, Indigenous Or Spiritual Observances (Riso)

Students requiring academic accommodation based on religious, indigenous or spiritual observances should follow the procedures set out in the RISO policy. Students should submit their request to their Faculty Office ***normally within 10 working days*** of the beginning of term in which they anticipate a need

for accommodation or to the Registrar's Office prior to their examinations. Students should also contact their instructors as soon as possible to make alternative arrangements for classes, assignments, and tests.

Courses with An On-Line Element

Some courses may use on-line elements (e.g. e-mail, Avenue to Learn (A2L), LearnLink, web pages, capa, Moodle, ThinkingCap, etc.). Students should be aware that, when they access the electronic components of a course using these elements, private information such as first and last names, user names for the McMaster e-mail accounts, and program affiliation may become apparent to all other students in the same course. The available information is dependent on the technology used. Continuation in a course that uses on-line elements will be deemed consent to this disclosure. If you have any questions or concerns about such disclosure, please discuss this with the course instructor.

Online Proctoring

Some courses may use online proctoring software for tests and exams. This software may require students to turn on their video camera, present identification, monitor and record their computer activities, and/or lock/restrict their browser or other applications/software during tests or exams. This software may be required to be installed before the test/exam begins.

Academic Integrity

You are expected to exhibit honesty and use ethical behaviour in all aspects of the learning process. Academic credentials you earn are rooted in principles of honesty and academic integrity.

It is your responsibility to understand what constitutes academic dishonesty.

Academic dishonesty is to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. This behaviour can result in serious consequences, e.g. the grade of zero on an assignment, loss of credit with a notation on the transcript (notation reads: "Grade of F assigned for academic dishonesty"), and/or suspension or expulsion from the university. For information on the various types of academic dishonesty please refer to the *Academic Integrity Policy*, located at https://secretariat.mcmaster.ca/university-policies-procedures- guidelines/

The following illustrates only three forms of academic dishonesty:

- plagiarism, e.g. the submission of work that is not one's own or for which other credit has been obtained.
- improper collaboration in group work.
- copying or using unauthorized aids in tests and examinations.

Authenticity / Plagiarism Detection

Some courses may use a web-based service (Turnitin.com) to reveal authenticity and ownership of student submitted work. For courses using such software, students will be expected to submit their work electronically either directly to Turnitin.com or via an online learning platform (e.g. A2L, etc.) using plagiarism detection (a service supported by Turnitin.com) so it can be checked for academic dishonesty.

Students who do not wish their work to be submitted through the plagiarism detection software must inform the Instructor before the assignment is due. No penalty will be assigned to a student who does not submit work to the plagiarism detection software. **All submitted work is subject to normal verification that standards of academic integrity have been upheld** (e.g., on-line search, other software, etc.). For more details about McMaster's use of Turnitin.com please go to the McMaster Office of Academic Integrity's webpage.

Conduct Expectations

As a McMaster student, you have the right to experience, and the responsibility to demonstrate, respectful and dignified interactions within all our living, learning and working communities. These expectations are described in the *Code of Student Rights & Responsibilities* (the "Code"). All students share the responsibility of maintaining a positive environment for the academic and personal growth of all McMaster community members, **whether in person or online**.

It is essential that students be mindful of their interactions online, as the Code remains in effect in virtual learning environments. The Code applies to any interactions that adversely affect, disrupt, or interfere with reasonable participation in University activities. Student disruptions or behaviours that interfere with university functions on online platforms (e.g. use of Avenue 2 Learn, WebEx or Zoom for delivery), will be taken very seriously and will be investigated. Outcomes may include restriction or removal of the involved students' access to these platforms.

Copyright and Recording

Students are advised that lectures, demonstrations, performances, and any other course material provided by an instructor include copyright protected works. The Copyright Act and copyright law protect every original literary, dramatic, musical and artistic work, **including lectures** by University instructors.

The recording of lectures, tutorials, or other methods of instruction may occur during a course. Recording may be done by either the instructor for the purpose of authorized distribution, or by a student for the purpose of personal study. Students should be aware that their voice and/or image may be recorded by others during the class. Please speak with the instructor if this is a concern for you.

Research Ethics -NA

Extreme Circumstances

The University reserves the right to change the dates and deadlines for any or all courses in extreme circumstances (e.g., severe weather, labour disruptions, etc.). Changes will be communicated through regular McMaster communication channels, such as McMaster Daily News, A2L and/or McMaster email.