HOMEWORK #2

Due: November 14 (Monday) by midnight

Submit all your files (i.e., your brief report, preferably in the PDF format, and all of your MATLAB m–files) via Email to the instructor. If you have many files, please compress them into a single archive. Avoid using spaces in file names (you can use an underscore instead).

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1. You are given a definite integral $I = \int_0^1 e^{-x}\,dx$. Approximate this integral numerically using:

   (a) the *trapezoidal* method

   (b) the *Gauss* method

   and $N = \{2,3,4,5\}$ grid points in the interval $[0,1]$. Compare the dependence of the relative error $|\frac{I_{approx}-I_{ex}}{I_{ex}}|$ on the number of grid points N in the two methods. Plot your results using logarithmic scale for the error.
   (5 points)

2. You are given a 2π–periodic function $f(x) = \frac{1}{1+\sin(x/2)^2}$. Using your favorite spectral differentiation approach calculate an approximation to the third derivative $\frac{d^3}{dx^3} f(x)$ of this function on the interval $\Omega = [0,2\pi]$ using $N = \{2^3,\ldots,2^{10}\}$ gridpoints. Employing a log–log plot, display the dependence of the ∞–norm of the differentiation error as a function of N. How to explain the behavior of the error for large values of N?
   (5 points)

   Remark — the ∞–norm of a function $g(x)$ is given by $\|g\|_\infty = \sup_{x\in\Omega} |g(x)|$.