MATH 745 — TOPICS IN NUMERICAL ANALYSIS

Time & Place — TBD
(tentatively scheduled for 11:00–12:30 on Mondays and 14:00–15:30 on Thursdays)
Organizational meeting will take place at 14:00 on Thursday, September 8, venue TBA)

Instructor: Dr. Bartosz Protas
Email: bprotas@mcmaster.ca
Office HH 326, Ext. 24116
Course Webpage: http://www.math.mcmaster.ca/~bprotas/MATH745c

---

Outline of the Course: The course will focus on techniques for numerical solution of Partial Differential Equations (PDEs). The objectives of the course are essentially twofold: first, provide students with an understanding of the deeper mathematical foundations for certain classical numerical methods which they should already be familiar with, and, secondly, introduce students to more advanced numerical methods for PDEs. The course will address both theoretical aspects, such as error and stability analysis, as well as certain implementation issues. The presented methods will be illustrated using well–known PDEs from mathematical physics. The specific topics that will be discussed include (optimistic variant):

1. Critical Review of Finite–Difference Methods
   (a) Discretization of differential operators; incorporation of boundary conditions
   (b) Accuracy and conditioning of numerical differentiation
   (c) Advanced numerical differentiation (complex step derivative, Padé schemes, compact finite differences)
2. Review of Approximation Theory
   (a) Functional analysis background (Hilbert spaces, inner products, orthogonality and orthogonal systems)
   (b) Best approximations
   (c) Interpolation theory
3. Spectral methods for PDEs
   (a) Differentiation in spectral space
   (b) Fourier and Chebyshev methods; fast transforms (FFT)
   (c) Application to nonlinear problems (pseudo–spectral methods, dealiasing)
4. Multiresolution methods for PDEs
   (a) Orthogonal wavelets
   (b) Discrete wavelet transform (DWT)
   (c) Multiresolution representation of functions

Primary Reference:

1. L. N. Trefethen, "Spectral Methods in Matlab", SIAM, (2000).

Supplemental Reference:

2. K. Atkinson and W. Han, "Theoretical Numerical Analysis: A Functional Analysis Framework", Springer (TAM 39), (2001).
3. J. P. Boyd, "Chebyshev and Fourier Spectral Methods, Second Edition (Revised)", Dover, (2001).
4. A. Cohen, "Numerical Analysis of Wavelet Methods", North-Holland, (2003)

In addition to the above references, sets of lecture notes and example MATLAB codes will be made available to students on the course webpage.

Prerequisites: Numerical Analysis at the undergraduate level (including numerical methods for ODEs and PDEs), Partial Differential Equations, basic programming skills in MATLAB

Grades: The final grades will be based on:

- two 20 min quizzes ($2 \times 10\% = 20\%$),
- two homework assignments ($2 \times 10\% = 20\%$),
- a take–home final project (60%).

Quiz dates and homework due dates will be announced during the first week of instruction. I reserve the right to alter your final grade, in which case, however, the grade may only be increased.