# QUIZ #1

10:30am, October 26 (Friday), 20 minutes, 10 points max
(no books, no notes)

Write your name and Email address on the top of this sheet
Write your answers on the reverse side and/or attach additional sheets as necessary.

---

1. You are given the following initial-value problem

$$\frac{d^2y}{dt^2} = -y, \quad y(0) = 1, \quad \frac{dy}{dt}(0) = 0.$$

Discretize it using

(a) Euler's explicit method

(b) leapfrog method

and state under what conditions on the time step $\Delta t$ these two schemes are stable. Use matrix notation.
*(5 points)*

2. Suppose you have a boundary-value problem in the form

$$\begin{aligned} &\frac{d^2y}{dx^2} - a\,y = f, \qquad \text{on } (-1,1) \\ &y(-1) = 0, \\ &y(1) = 0, \end{aligned}$$

where $a \in \mathbb{R}^+$ and $f : (-1,1) \to \mathbb{R}$ is a smooth function. Given the following set of $N$ orthonormal (with respect to the $L_2$ inner product with weight $w \equiv 1$) basis functions $\phi_1, \ldots, \phi_N : (-1,1) \to \mathbb{R}$ such that $\phi_j(\pm 1) = 0$, $j = 1, \ldots, N$, show how

(a) *the spectral Galerkin method*, and

(b) *the spectral collocation method*

can be derived as special cases of the Weighted Residual Method for the solution of the above problem. In both cases clearly exhibit the structure of the resulting algebraic problem.
*(5 points)*