QUIZ #2

10:30am, November 30 (Friday), 20 minutes, 10 points max
(no books, no notes)

Write your name and Email address on the top of this sheet
Write your answers on the reverse side and/or attach additional sheets as necessary.

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1. You are given the following partial differential equation

$$\begin{aligned} &\partial_t u + u\partial_x u - \nu\partial_x^2 u = f && \text{in } [-\pi,\pi]\times[0,T] \\ &u = u_0 && \text{in } [-\pi,\pi] \text{ at } t=0 \end{aligned}$$

   with periodic boundary conditions and where $f : [-\pi,\pi]\times[0,T]\rightarrow\mathbb{R}$. Propose a *spectral method* to solve this problem numerically. Discuss, in particular, the following issues:

   (a) choice of the time–integration schemes (first-order accurate, for simplicity) for the linear and nonlinear terms,

   (b) treatment of the nonlinear term which will ensure proper handling of the aliasing errors.

   (7 points)

2. The values of the function $f : [-1,1]\rightarrow\mathbb{R}$ are given on $N+1$ Gauss-Lobatto-Chebyshev points, i.e., $f_j = f\left(\cos(\frac{j\pi}{N})\right)$, $j = 0,\dots,N$. These values are used to construct an approximation of the function f in terms of its truncated Chebyshev series $f(x)\approx\sum_{k=0}^{N}\hat{f}_k\,T_k(x)$. Show how the *Chebyshev coefficients* $\hat{f}_k$ can be rapidly evaluated using a suitable form of the Fourier transform.
   (3 points)