QUIZ #1

9:30am, October 24 (Friday), 20 minutes, 8 points max

(no books, no notes)

Write your name and Email address on the top of this sheet
Write your answers on the reverse side and/or attach additional sheets as necessary.

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1. Consider a scalar initial-value problem in the form

$$\frac{dy}{dt} = \lambda\, y, \quad \text{with} \quad \mathrm{y}(0) = y_0, \tag{1}$$

where $\lambda \in \mathbb{C}$. A generic finite-difference method for the numerical solution of this problem can be written as

$$y_n = \sigma^n\, y_0 \tag{2}$$

for some $\sigma = \sigma(\lambda, h) \in \mathbb{C}$ where h is the step size. Under what conditions on $\sigma(\lambda, h)$ will the approximate solution defined by (2)

   (a) be stable,

   (b) conserve the amplitude of y?

(2 points)

2. Considering the problem defined above, what are the stability and conservation properties of

   (a) Euler's explicit method,

   (b) Euler's implicit method,

   (c) leapfrog method

depending on whether λ is purely real or purely imaginary?
(3 points)

3. Consider an element $\mathbf{x} \in \mathbb{R}^3$ and the subspace $H_2 \subset \mathbb{R}^3$ such that $H_2 = span\{\mathbf{e}_1, \mathbf{e}_2\}$, where $\mathbf{e}_1$ and $\mathbf{e}_2$ are the unit vectors associated with the x_1 and x_2 axes of the coordinate system. Find the element $\mathbf{a} \in H_2$, defined by the relation

$$\|\mathbf{x} - \mathbf{a}\| = \inf_{\mathbf{y} \in H_2} \|\mathbf{x} - \mathbf{y}\|, \tag{3}$$

in the following three cases

   (a) $\mathbf{x} = [1\ 2\ 3]^T$,

   (b) $\mathbf{x} = [1\ 2\ 0]^T$,

   (c) $\mathbf{x} = [0\ 0\ 3]^T$.

(3 points)